

## Clutter Filtering and Spectral Moment Estimation for Doppler Weather Radars Using Staggered Pulse Repetition Time (PRT)

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### ABSTRACT

In this paper, a new algorithm for the estimation of spectral parameters from the signal time series, collected using the staggered pulse repetition time (PRT) transmission in a Doppler weather radar, is presented. The algorithm uses the Fourier transform and a magnitude deconvolution procedure to reconstruct the signal spectrum, and then the spectral parameters are estimated from the reconstructed spectrum. There is a significant improvement in the variance of the spectral parameter estimates compared to previously published methods of processing staggered PRT sequences. Further, a novel spectral domain clutter filtering procedure allows 1) accurate velocity estimation even if the clutter-to-signal power ratio is as high as 40 dB and 2) does not incur the loss of velocity information in certain Doppler bands experienced by other clutter filtering techniques. With this algorithm, the staggered PRT technique becomes a practical contender for implementation on Doppler weather radars in the quest to increase the unambiguous velocity and range.

### 1. Introduction

One of the long-standing problems affecting Doppler weather radar performance has been the velocity and range ambiguity. For a uniform pulse repetition time (PRT) transmission, the unambiguous range  $r_a$  and velocity  $v_a$ , are governed by the equation  $v_a r_a = c\lambda/8$ , where  $c$  is the speed of light and  $\lambda$  is the wavelength. Further, in Doppler weather radars, velocity ambiguities can occur in the presence of overlaid echoes. In such cases, the overlaid signals must be separated prior to determining the proper spectral moments of each signal.

Several schemes to ameliorate the ambiguity problem have been proposed. Some of the important ones are (a) polarization coding (Doviak and Sirmans 1973), (b) random phase technique (Zrnić 1979; Laird 1981; Siggia 1983; Zrnić and Mahapatra 1985), (c) systematic phase coding (Sachidananda and Zrnić 1997), (d) staggered PRT technique (Zrnić and Mahapatra 1985), and (e) the method using two sampling rates (Sirmans et al. 1976).

The random phase technique attempts to solve the problem of echo overlay by whitening the spectrum of the unwanted overlaid signal so that it does not bias the

velocity estimate. However, the whitened unwanted signal power is equivalent to noise, which adversely affects the estimate variance of the desired signal. An adaptive filtering technique can be used to remove a significant part of the overlaid signal from the desired spectrum, thus improving the velocity estimates (Siggia 1983; Zrnić and Mahapatra 1985). This procedure nearly doubles the range in which signals can be recovered, for a given sampling rate (or unambiguous velocity) within certain limits of overlay power ratio and spectrum widths (Sachidananda et al. 1997). A more effective method for solving the overlay problem is the systematic phase coding (SZ codes) proposed by Sachidananda et al. (1997, 1998).

In addition to signal overlay, clutter filtering is another major problem that needs to be dealt with before ambiguity can be resolved. For uniformly sampled signals, the clutter filtering does not pose serious difficulties, but for the nonuniform sampling, as in the staggered PRT scheme, it is a major obstacle and the main reason that precluded implementation of the staggered PRT technique in practical weather radars. Although this technique expands both  $r_a$  and  $v_a$ , the associated estimators of mean velocity had relatively large variances because they included the difference between two estimates.

Promising approaches to deal with these deficiencies have recently emerged. For example, Weber and Chor-

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noboy (1993) and Chornoboy and Weber (1994) apply techniques for extrapolation of band-limited signals to extend the signal spectrum beyond the original measurement interval. These require testing to determine the correct frequency location where the band should be situated. Combined with a best-fit procedure, the method has been suggested for both dealiasing and clutter filtering of the signal on the ASR-9 airport surveillance radar, where the reported success in clutter filtering is aided by the block-staggered waveform (i.e., groups of contiguous samples within a dwell time). Although for a truly staggered PRT waveform, dealiasing based on extrapolation of band-limited signals is feasible, clutter filtering may not be satisfactory.

We seek to eliminate both the problem of poor moment estimates and clutter filtering of staggered PRT sequences by the spectral parameter estimation algorithm proposed in this paper. The main contribution of this paper is a new approach to clutter filtering and spectral parameter estimation, which, in addition to improving the estimate variance, enables effective filtering of the clutter without introducing undue bias error in the velocity estimate. The algorithm includes estimation of mean power, mean velocity, and spectrum width in the presence of ground clutter. The computations are carried out in the Fourier transform domain and, hence, need significantly more processing power than required for the pulse pair estimator. With present day processors, it is feasible to implement such an algorithm in a weather radar.

No attempt has been made to solve the overlay problem. The algorithm assumes that there is no overlaid signal. However, the staggered PRT technique allows selection of large, unambiguous range and velocity so that the extent of overlay can be significantly reduced by proper choice of PRTs.

## 2. The staggered PRT technique

In the staggered PRT technique (Zrnić and Mahapatra 1985) two different pulse spacings,  $T_1$  and  $T_2$ , are used alternately to obtain autocorrelations,  $R_1$  at lag  $T_1$ , and  $R_2$  at lag  $T_2$ . The velocity is estimated from the phase difference between the two using the formula

$$\hat{v} = \lambda \arg(R_1 R_2^*) / [4\pi(T_2 - T_1)]. \quad (1)$$

Thus, the difference in PRT,  $(T_2 - T_1)$ , determines the unambiguous velocity,  $v_a$ , which is given by

$$v_a = \pm \lambda / [4(T_2 - T_1)]; \quad T_1 < T_2. \quad (2)$$

Zrnić and Mahapatra (1985) suggest a testing procedure to estimate mean velocity and signal power for ranges within the time delay  $(T_1 + T_2)$ . In theory, this seems to be possible because the overlaid signal in any two consecutive samples is from two different ranges and therefore can be assumed to be uncorrelated. This implies that the expected value of the overlaid signal con-

tribution to the autocorrelation is zero. Thus, the unambiguous range becomes

$$r_a = c(T_1 + T_2)/2. \quad (3)$$

Equations (1) and (3) suggest the staggered PRT is equivalent to a uniform PRT  $= (T_2 - T_1)$  for the unambiguous velocity and a uniform PRT  $= (T_1 + T_2)$  for the unambiguous range, and each can be selected independently. This would be true in the absence of overlaid echoes. The practical utility of this scheme is questionable due to several limitations in the quality of estimates. The overlaid signal increases the variance of the estimates because it acts as white noise. Thus, the ratio of the overlaid signal powers is the equivalent signal-to-noise ratio (SNR), and for reasonable accuracy of the estimates, the unwanted signal has to be at least 3 dB below the desired signal power.

Let  $r_{a1} = cT_1/2$  and  $r_{a2} = cT_2/2$  so that  $r_a = r_{a1} + r_{a2}$ . If  $r_{a1}$  were chosen sufficiently large so that no echoes are received from ranges greater than  $r_{a1}$ , then the problem of overlaid echoes would be eliminated. At the 10-cm wavelength,  $r_{a1}$  would have to be larger than 460 km (for  $0.5^\circ$  in elevation), but this would degrade the variance of estimates. Thus the practical limit for  $r_{a1}$  is smaller, and the occasional presence of overlaid signals is expected.

It is shown by Zrnić and Mahapatra (1985) that the standard error in the velocity estimate increases as the ratio  $\kappa = T_1/T_2$  approaches unity, and a good choice is  $\kappa = 2/3$ . Thus, the unambiguous range and unambiguous velocity are indirectly tied in practice via the estimate accuracy. However, compared to the uniform PRT, it is possible to achieve much larger  $r_a$  and  $v_a$  because the limiting equation is  $v_a r_{a1} = \{\kappa/(1 - \kappa)\}c\lambda/8$  for the staggered PRT scheme.

A major problem with the staggered PRT scheme has been the clutter filtering. The nonuniform sampling aliases power from certain Doppler frequencies into the ground clutter frequency band around zero Doppler. Filtering the clutter also removes the signal power from these spectral coefficients and introduces phase perturbations at these frequencies, which bias the velocity estimate. Banjanin and Zrnić (1991) have investigated several methods of ground clutter filtering for the staggered PRT sequence. A scheme they proposed uses two filters sequentially such that overall filter coefficients are time varying. In the Doppler bands where the filter phase response is not linear, special decision logic corrects the velocity estimate. To overcome these obstacles, Chornoboy (1993) proposed block-staggered sampling and a least squares design of a filter matrix to achieve a desired frequency response. The added complexity of the pulse pattern enables an improved balance between magnitude and phase response so that Chornoboy (1993) achieved quite satisfactory results.

In the following, we present a very different approach to the clutter filtering and spectral moment estimation for staggered PRT transmission.

### 3. Processing procedure

In the proposed new approach we seek to reconstruct the spectrum of the weather signal from the staggered time series samples, that is, generate a time series with a uniform sampling period of  $T_u$ , and then estimate the spectral parameters from this reconstructed spectrum. This procedure allows estimation of the spectral moments with a much lower variance than the earlier methods. Further, a novel scheme for clutter filtering in the spectral domain is proposed that can achieve clutter suppression in excess of 40 dB and almost complete elimination of all spurious rejection bands in the  $\pm v_a$  interval. The result is a nearly linear phase response of the clutter filter, which has been eluding researchers so far. This is the most important feature of the clutter suppression scheme; it makes practical the use of staggered PRT in weather radars.

#### a. Reconstruction of the signal spectrum

The reconstruction procedure starts by representing the variable PRT sequence as a product of a uniformly sampled signal with a high PRT code that has unit elements that coincide with the positions of samples in the variable PRT sequence and zeroes otherwise. We select  $T_1$  and  $T_2$  such that they are integer multiples of some basic PRT  $T_u$ , so that  $T_1 = n_1 T_u$ , and  $T_2 = n_2 T_u$ , where  $n_1$  and  $n_2$  are integers; a good choice is  $n_2 = n_1 + 1$  and will be used in this paper. Thus,  $(T_2 - T_1) = (n_2 - n_1)T_u$  determines the unambiguous velocity,  $v_a$ , and  $r_{a1} = cT_1/2$ . Let  $g_i$ ,  $i = 0, 1, 2, 3, \dots, M - 1$ , be the alternately sampled (at time intervals  $T_1$  and  $T_2$ ) weather signal, and introduce zeros between  $g_i$  samples to produce a sequence  $v_i$  of length  $N = (n_1 + n_2)M/2$  with a uniform sampling period  $T_u$ . We call  $v_i$  the derived time series. Let  $c_i$  be a code sequence of length  $N$  obtained by replacing all the  $g_i$  samples in  $v_i$  by unity. For example,  $c_i = [1010010100 \dots]$  for  $\kappa = T_1/T_2 = 2/3$ . We can write the sample sequence  $v_i$  as a product of the sequence  $c_i$  and  $e_i$ , where  $e_i$  is the desired, but unknown, time series of the signal sampled at  $T_u$  intervals,

$$v_i = c_i e_i; \quad i = 0, 1, 2, 3, \dots, N - 1. \quad (4)$$

Therefore, the spectrum of  $v_i$  can be represented as a circular convolution ( $\star$ ) of the spectrum of the code  $c_i$  and the spectrum of the signal  $e_i$ ,

$$\text{DFT}(v_i) = \text{DFT}(c_i) \star \text{DFT}(e_i), \quad (5)$$

where DFT represents the discrete Fourier transform of the quantity in brackets. Our objective here is to reconstruct the signal  $e_i$  from the measured samples  $g_i$ .

We will begin this reconstruction in the spectral domain whereby the signal spectrum  $E_k = \text{DFT}(e_i)$ . Equation (5) can be written in matrix form as

$$\mathbf{V} = \mathbf{C}\mathbf{E}, \quad (6)$$

where  $\mathbf{V}$  and  $\mathbf{E}$  are  $(N \times 1)$  column vectors containing the DFT coefficients  $V_k$  and  $E_k$  of the corresponding time sequences,  $v_i$  and  $e_i$ ,  $\mathbf{C}$  is the convolution matrix (size:  $N \times N$ ) whose column vectors are cyclically shifted versions of  $C_k$ . To preserve the power in the spectrum, the convolution matrix  $\mathbf{C}$  is normalized such that each column vector is a unit vector (i.e., the norm of each column is unity). Note that by normalizing the column vectors we also normalize the row vectors. The convolution matrix is singular (its rank equals the number of staggered samples  $M$ ) and, hence, cannot be inverted to obtain  $E_k$ . If we discard the phases of the elements in the convolution matrix, the matrix becomes nonsingular. It may also be noted that it is sufficient to retrieve the magnitude of the spectral coefficients,  $\text{abs}\{E_k\}$ , from which it is possible to compute the autocorrelation  $R(T_u)$  and hence the mean velocity. Therefore, we attempt to retrieve  $\text{abs}\{\mathbf{E}\}$  using  $\text{abs}\{\mathbf{C}\}$ , which can be inverted.

In general,  $\text{abs}\{\mathbf{C}\mathbf{E}\} = \text{abs}\{\mathbf{C}\} \text{abs}\{\mathbf{E}\}$  does not hold because of the complex addition of the coefficients in the process of matrix multiplication. However, because the convolution matrix  $\mathbf{C}$  has only  $(n_1 + n_2)$  nonzero coefficients separated by  $N/(n_1 + n_2)$  coefficients for  $\kappa = n_1/n_2$  in each row (or column), we can show that if the signal spectrum is narrow so that its spread is less than  $N/(n_1 + n_2)$ , the complex addition does not occur in the process of convolution, and hence the equality is valid. Then the convolved matrix  $\mathbf{V}$  has  $(n_1 + n_2)$  replicas of the original spectrum  $\mathbf{E}$  (in the complex domain each replica has a specific complex multiplier), and these replicas do not overlap. Thus, we define a "magnitude deconvolution" as

$$\text{abs}\{\mathbf{E}\} = [\text{abs}\{\mathbf{C}\}]^{-1} \text{abs}\{\mathbf{V}\}, \quad (7)$$

where  $[\text{abs}\{\mathbf{C}\}]^{-1}$  is the magnitude deconvolution matrix. It is important to realize that this equation provides an accurate magnitude spectrum only under the condition that the signal spectrum is narrow. More precisely the total spread must be less than

$$2v_a/(n_1 + n_2) \quad (8)$$

in the velocity domain. This condition means the spectrum cannot alias on itself and is obtained from the average sampling rate  $2/(T_1 + T_2)$  for the staggered sequence. Although a wider spectrum is not reproduced faithfully in this procedure (because the spectral coefficients overlap in the spectrum  $V_k$ ), the nonideal reconstruction does not generally bias the velocity estimate but does affect its variance. If  $T_1$  and  $T_2$  are judiciously chosen, the criteria (8) can be nearly satisfied for most weather signals. For example, at a 10-cm wavelength and  $v_a = \pm 50 \text{ m s}^{-1}$ , (8) is nearly satisfied for spectrum widths,  $w < 6 \text{ m s}^{-1}$ . Nonetheless, it is important to note that the criteria is not satisfied exactly even for  $w = 4 \text{ m s}^{-1}$ .

Figure 1 illustrates the spectrum reconstruction of a simulated weather signal time series. A 10-cm wave-

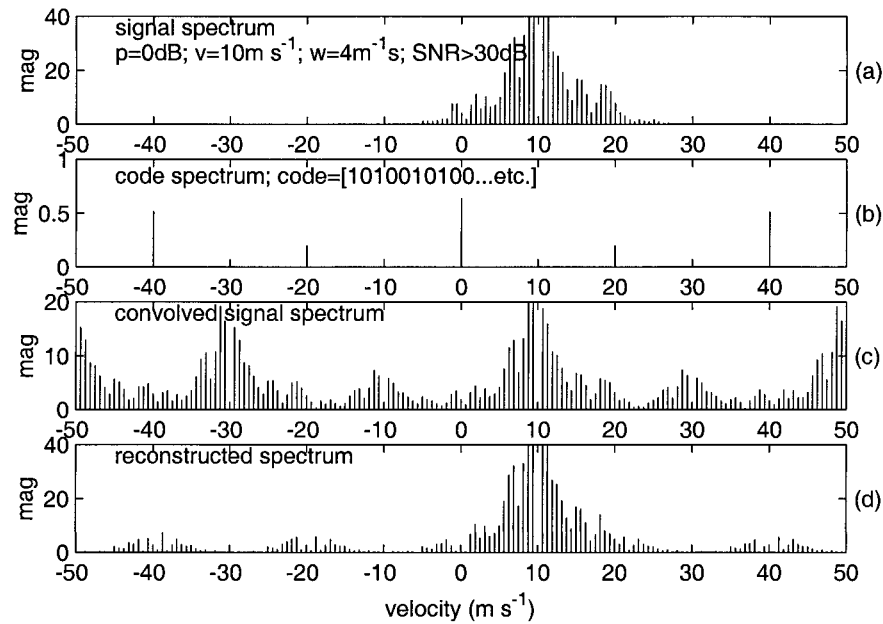


FIG. 1. Illustration of the magnitude domain deconvolution used to reconstruct the signal spectrum from the staggered PRT samples. Mean velocity is  $10 \text{ m s}^{-1}$ , spectrum width is  $4 \text{ m s}^{-1}$  and the SNR is  $>30 \text{ dB}$ . The number of uniformly spaced samples  $N$  in the derived time series is 160, the stagger ratio is  $\frac{2}{3}$ , the number of uniformly spaced samples within  $T_1$  is 2, and within  $T_2$  it is 3.

length is assumed, and the magnitude spectrum of the complete time series (signal sampled at a uniform sampling period of  $T_u = 0.5 \text{ ms}$ ) is shown in Fig. 1a. The magnitude spectrum of the code for staggered PRT sampling with  $\kappa = \frac{2}{3}$  (code is 1010010100 . . .) is plotted in Fig. 1b. If staggered PRT sampling were used ( $T_1 = 1 \text{ ms}$ ,  $T_2 = 1.5 \text{ ms}$ ), the spectrum of the derived uniform time series would be as given in Fig. 1c, which is the convolution of the first two spectra. The last spectrum (Fig. 1d) is the reconstructed spectrum from the staggered PRT samples using the magnitude deconvolution procedure. As mentioned earlier, for  $w = 4 \text{ m s}^{-1}$ , the spectrum does not exactly satisfy criteria (8); hence, a few residual spectral coefficients remain throughout the spectrum, but they would not bias the mean velocity because of their symmetry about the mean Doppler shift.

Once the magnitude spectrum is obtained, the spectral domain equivalent of the pulse pair algorithm can be used to estimate the mean power  $p$ , mean velocity  $v$ , and spectrum width  $w$ . The autocorrelation  $R(T_u)$  can be expressed as

$$R(T_u) = \sum_{k=0}^{N-1} |E_k|^2 e^{j2\pi k T_u / N}. \quad (9)$$

The velocity is obtained from the phase of  $R(T_u)$ . The conventional pulse pair algorithm can be used for mean power and width estimation. If the power is estimated from the reconstructed time sequence or spectrum, a multiplicative correction factor  $(n_1 + n_2)/2$  should be applied to get the correct result.

An important advantage of this procedure is that the estimate variances are much smaller than the ones obtained from product (1) of  $R(T_1)$  and  $R^*(T_2)$ . Because the procedure is nonlinear, it was deemed that derivations of theoretical expressions for the variance of the estimates would be time consuming and the results only approximately valid. Therefore, we chose simulations to obtain the standard errors of the velocity estimate,  $\text{sd}(v)$ . These are plotted, as a function of the spectrum width, in Fig. 2 for (a) the pulse pair velocity estimator that uses the complete time series with sampling period of  $T_u$ , (b) the proposed method of reconstructing the spectrum from staggered PRT samples, and (c) the pulse pair velocity estimator that uses the phase of  $[R(T_1)/R(T_2)]$ . The proposed method produces significantly better estimates, and its  $\text{sd}(v)$  compares very well with that of the pulse pair estimator on a complete time series. The spectrum reconstruction is good for widths up to about  $6 \text{ m s}^{-1}$ .

Because the spectrum is reconstructed, it is possible to devise a width estimator that has a smaller standard error than what ensues from  $\log[|R(T_1)/R(T_2)|]$ . For example the estimator based on  $\log[|R(0)/R(T_u)|]$  [Eq. (6.37) in Doviak and Zrnić (1993)] yields at least a threefold decrease in the standard error of the estimate—almost the same as the pulse pair estimator using the complete time series (Fig. 3). For the presented results the time series data had the inherent rectangular window. For low spectral widths the width estimates had a positive bias, which can be eliminated by applying a tapered

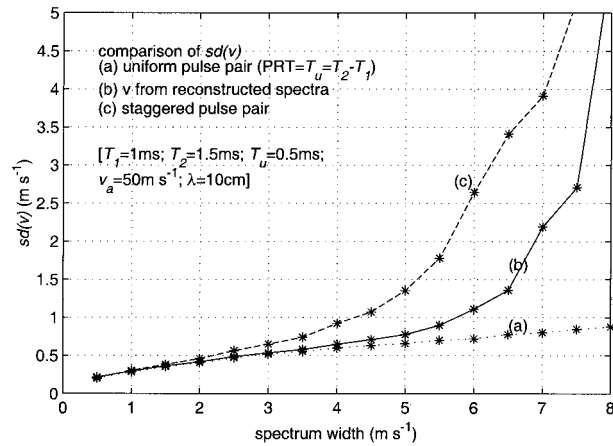


FIG. 2. Standard deviation of the velocity estimates as a function of spectrum width of the signal for (a) the pulse pair algorithm on a complete time series of uniformly spaced samples, (b) the proposed algorithm that reconstructs the spectrum, and (c) the staggered pulse pair algorithm based on  $R(T_1)R^*(T_2)$ .

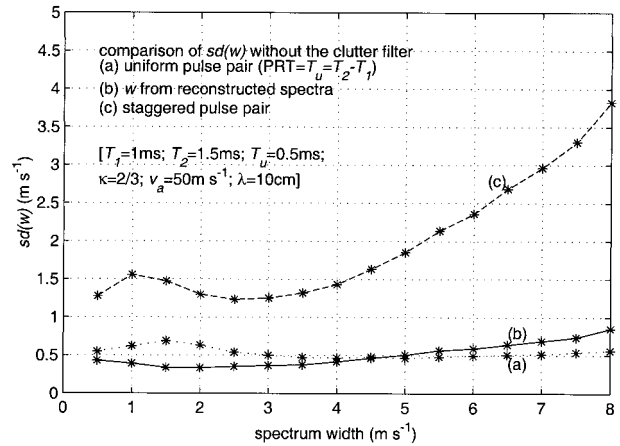


FIG. 3. The standard deviation of the spectrum width estimates as a function of the spectrum width of the signal for (a) the pulse pair estimator applied to the complete time series samples, (b) the pulse pair estimator applied to the reconstructed spectrum, and (c) the estimator based on  $\log[|R(T_1)/R(T_2)|]$ .

window (e.g., von Hann) or by choosing a different estimator.

*b. Ground clutter filtering*

The clutter filtering is carried out in the spectral domain before the signal spectrum is reconstructed using the deconvolution procedure described in section 3a. To prevent clutter spreading in the frequency domain and thus achieve better clutter rejection, the time series data must be multiplied by a suitable window function (von Hann window is used here). The spectrum of the ground clutter is assumed to have narrow width and is centered on zero Doppler velocity. Because the spectrum of the derived time series  $V_k$  is the convolution of the code spectrum  $C_k$  and the signal spectrum  $E_k$ , it will have weighted replicas of the clutter spectrum centered on each of the nonzero spectral lines of the code spectrum. The weights are the spectral coefficients of the code spectrum. An example of the convolved spectrum is in Fig. 4c for  $\kappa = T_1/T_2 = 2/3$ . The method (to be described) is not confined to  $\kappa = 2/3$ ; this stagger ratio is used only as an example to illustrate the clutter-filtering procedure. Because the code spectrum in this case has five nonzero spectral coefficients (see Fig. 1b), the power from each of the clutter spectral coefficients is spread over five spectral coefficients separated by  $N/5$  coefficients in the convolved spectrum. Specifically, power in the  $i$ th spectral coefficient is distributed over spectral coefficients at  $i, i + M/2, i + M, i + 3M/2,$  and  $i + 2M,$  cyclically (i.e., if any of these indices exceeds  $N$ , subtract  $N$  from the number). This type of modulation also affects the weather signal so that its spectrum too would be replicated into five bands. The modulation could cause weather and clutter that did not overlap in the unmodulated spectrum to overlap after modulation. In general, the power in each spectral coefficient is spread into only

$(n_1 + n_2)$  coefficients separated by  $N/(n_1 + n_2)$  coefficients. Therefore, we can take  $(n_1 + n_2)$  coefficients at a time and try to restore the spectrum. This is done by rearranging the matrices as described below.

Consider an example with parameters  $\kappa = 2/3, M = 64$  for which  $N = 160$ . Because the convolution matrix

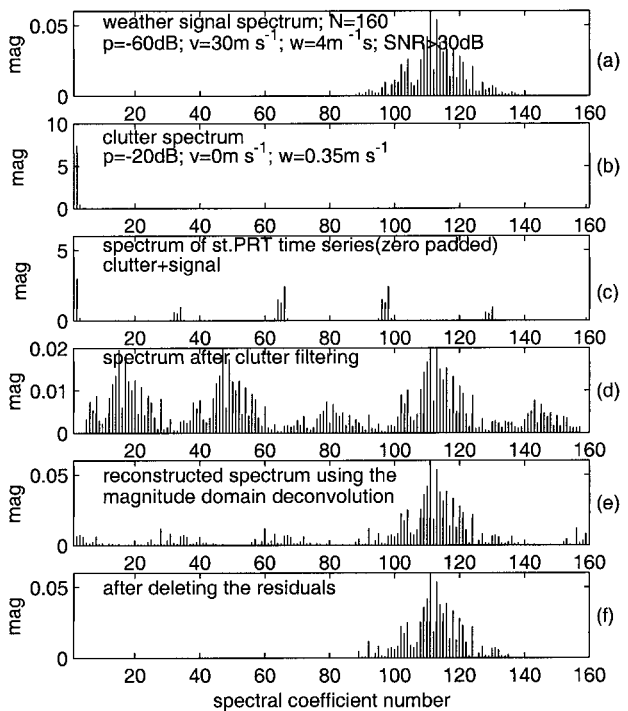


FIG. 4. An illustration of the clutter-filtering procedure in the proposed algorithm. The various spectra and associated parameters are indicated in each graph. Note that the scales on the ordinates are not equal.

has only five nonzero coefficients in each row (or column), we can recast the matrix (6) equation in the form,

$$\mathbf{V}_r = \mathbf{C}_r \mathbf{E}_r, \quad (10)$$

where subscript  $r$  is used to represent rearranged matrices, which are given by

$$\mathbf{V}_r = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 & \mathbf{V}_3 & \cdots & \mathbf{V}_{32} \\ \mathbf{V}_{33} & \mathbf{V}_{34} & \mathbf{V}_{35} & \cdots & \mathbf{V}_{64} \\ \mathbf{V}_{65} & \mathbf{V}_{66} & \mathbf{V}_{67} & \cdots & \mathbf{V}_{96} \\ \mathbf{V}_{97} & \mathbf{V}_{98} & \mathbf{V}_{99} & \cdots & \mathbf{V}_{128} \\ \mathbf{V}_{129} & \mathbf{V}_{130} & \mathbf{V}_{131} & \cdots & \mathbf{V}_{160} \end{bmatrix}, \quad (11)$$

$$\mathbf{C}_r = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_{129} & \mathbf{C}_{97} & \mathbf{C}_{65} & \mathbf{C}_{33} \\ \mathbf{C}_{33} & \mathbf{C}_1 & \mathbf{C}_{129} & \mathbf{C}_{97} & \mathbf{C}_{65} \\ \mathbf{C}_{65} & \mathbf{C}_{33} & \mathbf{C}_1 & \mathbf{C}_{129} & \mathbf{C}_{97} \\ \mathbf{C}_{97} & \mathbf{C}_{65} & \mathbf{C}_{33} & \mathbf{C}_1 & \mathbf{C}_{129} \\ \mathbf{C}_{129} & \mathbf{C}_{97} & \mathbf{C}_{65} & \mathbf{C}_{33} & \mathbf{C}_1 \end{bmatrix}, \quad \text{and} \quad (12)$$

$$\mathbf{E}_r = \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 & \mathbf{E}_3 & \cdots & \mathbf{E}_{32} \\ \mathbf{E}_{33} & \mathbf{E}_{34} & \mathbf{E}_{35} & \cdots & \mathbf{E}_{64} \\ \mathbf{E}_{65} & \mathbf{E}_{66} & \mathbf{E}_{67} & \cdots & \mathbf{E}_{96} \\ \mathbf{E}_{97} & \mathbf{E}_{98} & \mathbf{E}_{99} & \cdots & \mathbf{E}_{128} \\ \mathbf{E}_{129} & \mathbf{E}_{130} & \mathbf{E}_{131} & \cdots & \mathbf{E}_{160} \end{bmatrix}. \quad (13)$$

The convolution matrix  $\mathbf{C}$  is modified by deleting all the rows and columns containing zero coefficients, which reduces it to a  $5 \times 5$  matrix;  $\mathbf{C}_r$ , and the other two vectors are rearranged into a  $5 \times 32$  matrix each. In the complete time series spectrum  $E_k$ , the clutter is confined to the first and last few coefficients ( $k = 1, 2, \dots, q$  and  $N - q - 2, \dots, N$ ). But the matrix  $\mathbf{V}_r$  has clutter power spread over the first few and the last few columns. The matrix  $\mathbf{C}_r$  is singular (its rank is 3) and hence cannot be inverted; its rows are normalized so that the sum of the magnitudes squared of the elements in a row is unity.

To filter the clutter from the first few columns of  $\mathbf{V}_r$ , first estimate the complex amplitude of the clutter coefficient in the column vector (which also might contain the signal component). This is accomplished by taking the inner product of that column with the complex conjugate of the first column vector of the matrix  $\mathbf{C}_r$  (each column is normalized to unity) to produce the amplitude of the clutter vector in that column of  $\mathbf{V}_r$ . This amplitude is multiplied by the first column vector of  $\mathbf{C}_r$  to reconstruct the column clutter vector, which is then subtracted from the corresponding column vector of the matrix  $\mathbf{V}_r$ . This process is carried out for the first few columns containing clutter. Similarly, the clutter from the last few columns is removed using the last column vector of  $\mathbf{C}_r$  in place of the first. This complete operation can be written in matrix notation as

$$\mathbf{V}_f = \mathbf{V}_r - \mathbf{C}_{f1} \mathbf{V}_r \mathbf{I}_{f1} - \mathbf{C}_{f2} \mathbf{V}_r \mathbf{I}_{f2}, \quad (14)$$

where  $\mathbf{V}_f$  is the filtered spectrum matrix,  $\mathbf{C}_{f1}$  and  $\mathbf{C}_{f2}$  are

the clutter filter matrices, and  $\mathbf{I}_{f1}$  and  $\mathbf{I}_{f2}$  are matrices that select the columns to be filtered. The clutter filter matrices are given by

$$\mathbf{C}_{f1} = \mathbf{C}_1 \mathbf{C}_1^{t*} \quad \text{and} \quad (15a)$$

$$\mathbf{C}_{f2} = \mathbf{C}_2 \mathbf{C}_2^{t*}, \quad (15b)$$

where  $\mathbf{C}_1$  is the first column vector of  $\mathbf{C}_r$ ,  $\mathbf{C}_2$  is the last column vector of  $\mathbf{C}_r$ , the symbol  $(*)$  indicates conjugate, and the superscript  $t$  stands for the transpose. The matrix  $\mathbf{I}_{f1}$  is a  $M/2 \times M/2$  diagonal with the first  $q$  elements unity and the rest zeroes. Similarly, the matrix  $\mathbf{I}_{f2}$  is a  $M/2 \times M/2$  diagonal with the last  $(q - 1)$  elements unity and the rest zeroes. These matrices determine the clutter filter width. Note that  $(2q - 1) = n_c$  is the number of columns containing the clutter we wish to filter, or it is the clutter filter width in terms of the number of spectrum coefficients. This number will depend on the expected spectrum width of the clutter.

The matrix operation (14) produces the filtered signal matrix  $\mathbf{V}_f$ . Then,  $\mathbf{V}_f$  is rearranged in a single column real vector  $\mathbf{V}_s$ , in which only the magnitudes of the elements are used. To reconstruct the spectrum of the weather signal, the magnitude deconvolution procedure, as explained earlier [see Eq. (7)], is then applied.

Because the columns of  $\mathbf{C}_r$  are not orthogonal, clutter filtering also filters a small part of the signal power from the corresponding spectral coefficients. This results in residual spectral coefficients at the locations where clutter was present. However, the amplitudes of these residuals are comparable to or less than the signal coefficients. The residuals do not produce appreciable bias error in the mean velocity because (a) they are small if the signal spectrum and the clutter spectrum do not overlap in the spectrum of staggered PRT sequence, and (b) if there is overlap, the residuals (which now can be comparable to the signal coefficients) are nearly symmetrically placed about the mean velocity. This is the most important feature of our clutter-filtering procedure, and it leads to an equivalent clutter filter transfer function, which does not produce any loss of velocity information.

The mean velocity is computed from the phase of the autocorrelation  $R(T_u)$  as explained earlier. The spectrum width estimate would be biased because of the residuals. This bias is removed by deleting all the spectral coefficients outside  $(2N/p - 2q + 1)$  coefficients centered on the mean velocity and then computing the spectrum width.

Figure 4 is an illustration of the clutter-filtering procedure using a simulated signal; the clutter-to-signal power ratio (CSR) is 40 dB. The complete weather signal sampled with a uniform period of  $(T_2 - T_1)$  has a spectrum as shown in Fig. 4a, which is what we want to reconstruct from the staggered PRT samples. The complete clutter spectrum is in Fig. 4b, and the composite spectrum of the signal plus clutter computed from the staggered PRT samples (or the derived uniform se-

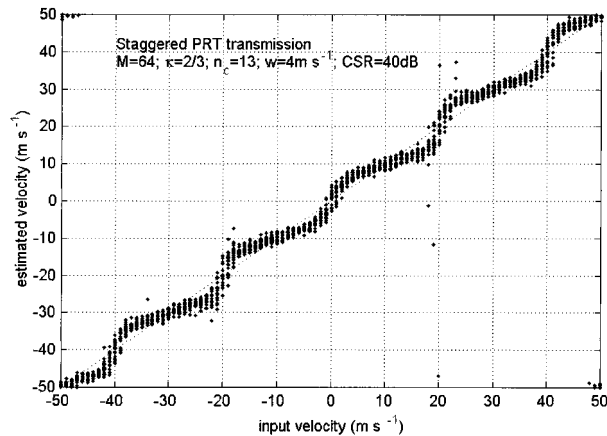


FIG. 5. Input velocity vs the mean (solid curve) of the estimated velocities from 20 simulations using the proposed algorithm. The dots about the mean are results from simulations, and  $n_c$  is the number of removed clutter spectral coefficients.

quence) is shown in Fig. 4c. Note that the clutter is spread over five coefficients across the spectrum. The weather signal is not visible because it is 40 dB below the clutter. The spectrum after the clutter is filtered is given in Fig. 4d. The magnitude deconvolution reconstructs the original spectrum (Fig. 4e) with some residuals at the locations where clutter was present. These are deleted in the last spectrum (Fig. 4f), which looks nearly the same as the original except for some coefficients at the tails of the spectrum.

This method was tested using simulated staggered PRT weather signals with an unambiguous velocity interval of  $\pm 50 \text{ m s}^{-1}$ . Figure 5 shows the mean estimated velocity as a function of the input velocity for one specific set of parameters indicated in the figure. At each input velocity 20 simulations were run, and the estimated velocities are shown as points. The continuous graph is the mean of the 20 estimated values of the velocity. All the previously published methods of clutter-filtering produce clutter filter notches at velocities  $\pm 20 \text{ m s}^{-1}$  and  $\pm 40 \text{ m s}^{-1}$  (locations where the velocity equals  $n\lambda/[2(T_1 + T_2)]$ ), in addition to the notch at zero Doppler. Note that the proposed method accurately recovers the velocity at these values. There is a small bias between two adjacent notch locations because of the residual coefficients, which, however, are within  $\pm 3 \text{ m s}^{-1}$ , and hence would be acceptable for some applications. This bias is a function of clutter filter width. The width of the clutter filter required for effective filtering depends on the CSR and the spectrum width of the clutter.

The clutter filtering also removes a small part of the signal power wherever the signal and clutter spectra overlap. This bias error is maximum if the mean velocity is  $\pm 5$ ,  $\pm 15$ ,  $\pm 25$ ,  $\pm 35$ , or  $\pm 45 \text{ m s}^{-1}$ , and it is zero if the velocity is  $\pm 10$ ,  $\pm 20$ ,  $\pm 30$ ,  $\pm 40$ , or  $\pm 50 \text{ m s}^{-1}$ . In Fig. 6 the bias and the standard deviation of the velocity estimate are shown for the same set of parameters. There

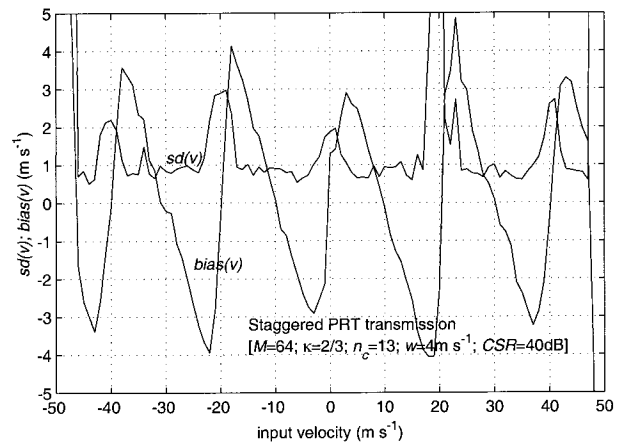


FIG. 6. Bias and standard errors of the velocity estimates obtained with the proposed algorithm.

is also some increase in the standard error wherever the signal spectrum and the clutter spectrum overlap (i.e., at velocities  $\pm 20 \text{ m s}^{-1}$  and  $\pm 40 \text{ m s}^{-1}$ ) (Sachidananda et al. 1999).

Similarly to the velocity estimates the clutter filter causes bias in the spectrum width estimates. The bias in the velocity and width estimates can be removed by additional processing, which we are currently evaluating.

For signals with larger spectrum width ( $w > 6 \text{ m s}^{-1}$ ), the reconstruction of the spectra is not accurate; hence, there is an increase in the bias as well as in the variance of the velocity estimates. The width is underestimated because the signal components are also removed in the process of deleting the residuals in the last step in the algorithm.

The method is general and can be adapted to any value of  $\kappa$ , but the velocity recovery and the clutter-filtering performance depend on several factors such as the number of zeros required to be inserted to derive a uniform PRT sequence in relation to the available staggered PRT samples. The largest ratio  $M/N$  is obtained for  $\kappa = 2/3$ , and it is the same value for which Zrnić and Mahapatra (1985) analyzed the variance of the velocity estimator (1). Also, Banjanin and Zrnić (1991) devised an elaborate, yet less effective, clutter-filtering procedure for this  $\kappa$ .

### c. The algorithm

The spectral parameter estimation algorithm for the Doppler weather radar with staggered PRT transmission scheme is as follows:

- Transmit the pulses at PRTs  $T_1$  and  $T_2$  alternately. The number of samples collected using the staggered PRT scheme is  $M$  for each range gate. The PRTs are selected such that  $T_1 = n_1 T_u$ , and  $T_2 = n_2 T_u$ .
- Insert zeroes in the time series to derive a uniformly

sampled sequence  $v_i$  of length  $N = pM/2$ ;  $p = (n_1 + n_2)$ .

- Multiply the time series by window weights (von Hann window gives very good performance).
- Compute the DFT of the time series and rearrange the spectral coefficients into a  $(p \times M/2)$  size matrix  $\mathbf{V}_r$ . The spectral coefficients are arranged row-wise to form the matrix.
- Compute the DFT of the code corresponding to the chosen value of  $\kappa$  and extract only the  $p$  nonzero coefficients of the spectrum. Arrange these coefficients in a column vector,  $\mathbf{C}_1$ . Normalize the coefficients such that  $\mathbf{C}_1$  is a unit vector.
- Form matrix  $\mathbf{C}_2$  by shifting the first coefficient of  $\mathbf{C}_1$  to the last.
- Compute the clutter filter matrices  $\mathbf{C}_{f1}$  and  $\mathbf{C}_{f2}$  using (15a) and (15b),

$$\mathbf{C}_{f1} = \mathbf{C}_1 \mathbf{C}_1^* \quad \mathbf{C}_{f2} = \mathbf{C}_2 \mathbf{C}_2^*.$$

- Determine the clutter filter width,  $(2q - 1)$ , in terms of the number of spectral coefficients, and form the  $(M/2 \times M/2)$  diagonal matrices,  $\mathbf{I}_{f1}$  and  $\mathbf{I}_{f2}$ , which select the columns that contain clutter signal.
- Filter the clutter by performing the matrix multiplication and subtraction indicated in (14),

$$\mathbf{V}_f = \mathbf{V}_r - \mathbf{C}_{f1} \mathbf{V}_r \mathbf{I}_{f1} - \mathbf{C}_{f2} \mathbf{V}_r \mathbf{I}_{f2}.$$

- Rearrange the matrix  $\mathbf{V}_f$  into a column matrix  $\mathbf{V}$ , compute  $\text{abs}\{\mathbf{V}\}$ , and then carry out the magnitude domain deconvolution given by (7);

$$\text{abs}\{\mathbf{E}\} = [\text{abs}\{\mathbf{C}\}]^{-1} \text{abs}\{\mathbf{V}\}.$$

- Compute the autocorrelation  $R(T_u)$  using (9),

$$R(T_u) = \sum_{k=1}^{N-1} |E_k|^2 \exp(j2\pi k T_u / N),$$

and the mean velocity from the phase of  $R(T_u)$ ,

$$v_m = (v_a / \pi) \arg\{R(T_u)\}; \quad v_a = \lambda / (4T_u).$$

- Compute the extent of the residual spectrum to be deleted based on the value of estimated velocity,  $v_m$ .
- Delete the residual coefficients and estimate the spectrum width and mean power.

#### 4. Conclusions

A new spectral domain processing technique is presented for filtering the clutter and estimating the spectral parameters of a weather signal; the scheme is meant for Doppler weather radars that transmit a staggered PRT sequence to mitigate velocity and range ambiguities. Although we have demonstrated it on a staggered PRT sequence, our new procedure of both spectral reconstruction and clutter filtering is suitable for any variable PRT scheme. It starts by representing the variable PRT sequence as a product of a uniformly sampled signal with a high PRT code that has unit elements that co-

incide with the positions of samples in the variable PRT sequence and zeroes otherwise. Then, clutter filtering in the spectral domain and magnitude deconvolution are used to restore the spectrum in an extended unambiguous interval.

Clutter filtering entails vectorial subtraction in the frequency bands where the clutter is spread by the stagger modulation. This subtraction does not produce spurious rejection bands in the Doppler spectrum. Magnitude domain deconvolution restores, with little distortion, the weather signal in the extended velocity interval. This makes it feasible to estimate spectral moments with much lower standard errors than possible with earlier methods. We have demonstrated the method on a staggered PRT sequence with the stagger ratio of  $2/3$ . Besides being capable of filtering 40 dB of clutter while preserving the mean velocity estimates, the method also reduces the standard errors of estimates and, thus, is preferred even if clutter is absent.

The spurious rejection bands in the clutter filter were the main obstacle preventing implementation of the staggered PRT scheme in practical radars. With this new method of clutter filtering, it becomes practical to use the staggered PRT scheme and achieve increased unambiguous velocity and range (D. Zrnić and M. Sachidananda, unpublished manuscript).

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