Unambiguous Range Extension by Overlay Resolution in Staggered PRT Technique

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ABSTRACT

In this paper a method is presented for separating overlaid echoes in a Doppler weather radar that uses a staggered pulse repetition time (PRT) transmission scheme to mitigate the effects of range–velocity ambiguities. In the standard staggered PRT technique, the PRT alternates between two values, T_1 and T_2; ($T_1 < T_2$) and the unambiguous range corresponds to the shorter PRT (T_1). If the weather extends up to the range corresponding to the longer of the two PRTs, some echoes from the short PRT will arrive at the receiver after the transmission of the long PRT. Therefore, echoes from the short and long PRTs that arrive at the same time but originate from range locations spaced $cT_1/2$ apart will be overlaid. An algorithm is developed to separate the overlaid echoes and estimate the spectral moments of both the overlaid signals. This effectively increases the unambiguous range to $cT_2/2$, corresponding to the longer PRT. Conditions for which the algorithm could be applied are described, and a strategy on how to use it in a range–velocity ambiguity mitigation scheme is outlined. The method of overlay resolution is tested on simulated time series. These tests illustrate the capability to separate the overlaid echoes and identify characteristics of weather signals for which the algorithm is expected to perform well.

1. Introduction

For a Doppler weather radar with a uniform pulse repetition time (PRT) transmission, the unambiguous range r_a and the unambiguous velocity υ_a are related by $\upsilon_a r_a = c\lambda/8$, where c is the speed of light and λ is the wavelength. Further, the velocity estimate can be erroneous if the echoes are overlaid, even if the actual velocity is less than υ_a. In such cases, the overlaid signals must be separated prior to estimating the spectral moments of the two overlaid signals. The staggered PRT technique is an approach to the range–velocity ambiguity resolution.

Range and velocity ambiguity affecting the Doppler weather radar has been a topic of investigation by several researchers. Some of the solutions proposed for mitigating the ambiguity problem are (a) polarization coding (Doviak and Sirmans 1973), (b) random phase technique (Zrnic 1979; Laird 1981; Siggia 1983; Zrnic and Mahapatra 1985), (c) systematic phase coding (Sachidananda et al. 1997, 1998; Sachidananda and Zrnic 1999), and (d) staggered PRT technique (Sirmans et al. 1976; Zrnic and Mahapatra 1985; Sachidananda et al. 1999; Sachidananda and Zrnic 2000).

The random phase technique attempts to solve the problem of echo overlay by whitening the spectrum of the unwanted overlaid signal so that it does not bias the velocity estimate. Both generation of random phase and whitening of out-of-trip signals are automatic on radars with magnetron transmitters. Adaptive filtering techniques have been proposed to remove a significant part of the overlaid signal from the desired signal spectrum, thus improving the velocity estimates (Siggia 1983; Zrnic and Mahapatra 1985). Such procedures double the unambiguous range for a given sampling rate (or unambiguous velocity) within certain limits of overlay power ratio and spectrum widths (Sachidananda et al. 1997). A more effective method for solving the overlay problem is the systematic phase coding (SZ codes) suggested by Sachidananda and Zrnic (1999).

The staggered PRT technique (Zrnic and Mahapatra 1985) is one of the suggested methods to increase the unambiguous velocity without compromising the unambiguous range. In this technique, the PRT alternates between two values, T_1 and T_2; ($T_1 < T_2$), so that autocorrelations at these two lags can be computed from the return signal time series. Using these two autocorrelation estimates, $R_1(T_1)$ and $R_2(T_2)$, it is possible to estimate mean velocity over an unambiguous interval corresponding to the difference in sampling times, ($T_2 - T_1$). Although an unambiguous range of $c(T_1 + T_2)/2$ is

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theoretically possible, because the overlaid signals in alternate samples come from different ranges and, hence, are uncorrelated, its practical utility is limited due to the large variance of the estimates. This essentially restricts the practical unambiguous range to $r_{a1} = cT_1/2$ (corresponding to T_1) for which the overlay can be avoided. This technique appears to be a good solution for the ambiguity problem, but until recently, methods were lacking to filter the ground clutter because of the nonuniform sampling; hence, the technique has not been implemented on operational weather radars.

Banjanin and Zrnic (1991) have investigated several methods of ground clutter filtering for the staggered PRT sample sequences. Nonuniform sampling perturbs the amplitude and phase characteristics of clutter filters, causing loss of Doppler information in certain regions within the unambiguous interval. To overcome these problems, Chornoboy (1993) proposed block-staggered sampling which enables an improved balance between magnitude and phase response and, thus, achieves quite satisfactory results, but the transmission scheme is more complex than the simple two PRT staggered sampling.

Sachidananda and Zrnic (2000) have developed a spectral domain clutter filtering technique for the staggered PRT sequence. The technique is developed assuming no overlay; i.e., the shorter of the two PRTs is selected large enough to avoid overlaid echoes. This method along with a bias correction procedure (Sachidananda et al. 1999) is very effective for recovering spectral moments of weak signals in the presence of clutter-to-signal power ratio as large as 50 dB. But the assumption of no overlay limits the unambiguous range to $r_{a1} = cT_1/2$. Thus, while selecting the PRTs, it is necessary to ensure that no weather is observed beyond r_{a1}.

If there are returns from ranges beyond r_{a1}, but less than $r_{a2} = cT_2/2$ (corresponding to the longer of the two PRTs), then overlay occurs in alternate samples of the time series sequence for some of the range gates. Henceforth, we refer to this as a *one-overlay* situation. If the one overlay could be resolved, it would effectively extend the practical unambiguous range from r_{a1} to r_{a2}. Herein, we explore the possibility to resolve the one-overlay echoes. We propose an algorithm to resolve the two overlaid signals and estimate their spectral moments, and then we evaluate the capabilities and limitations of the proposed algorithm.

2. The staggered PRT technique

In the staggered PRT technique (Zrnic and Mahapatra 1985), the transmitted sequence consists of two alternating pulse spacings, T_1 and T_2; ($T_2 > T_1$). Then, alternate pairs of return samples are used to compute autocorrelation estimates, R_1 at lag T_1 and R_2 at lag T_2. The velocity can be estimated from the phase difference between the two using the formula (Doviak and Zrnic 1993)

$$\hat{v} = \lambda \arg(R_1 R_2^*)/[4\pi(T_2 - T_1)]. \tag{1}$$

Thus, the difference in PRTs, $(T_2 - T_1)$ determines the unambiguous velocity, v_a, for the staggered PRT technique and is given by

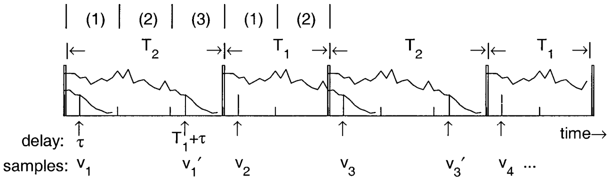
$$v_a = \lambda/[4(T_2 - T_1)]; \qquad T_1 < T_2. \tag{2}$$

A better estimate can be obtained from a weighted average of the two dealised velocities, whereby the weights are functions of variances and covariance of the two variables. Because the variance of v_1 (for $T_1 < T_2$ as is the case here) is much smaller than the variance of v_2, it turns out that a practical (close to minimum mean-square error) estimate is obtained as follows. Use R_1 for computing an aliased velocity v_1 and include the velocity from R_2 to dealias v_1 over an unambiguous interval $\pm v_a$ (Sachidananda et al. 2001). This approach gives a better velocity estimate than (1).

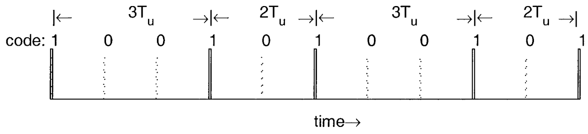
It is shown by Zrnic and Mahapatra (1985) that the standard error in the velocity estimate increases as the ratio, $\kappa = T_1/T_2$, approaches unity, and a good choice is $\kappa = 2/3$. Thus, the unambiguous range and the unambiguous velocity are indirectly restricted via the estimate accuracy. Nonetheless, compared to the uniform PRT, it is possible to achieve a larger r_a and v_a because the limiting equation is $v_a\, r_{a2} = [1/(1 - \kappa)]\, c\lambda/8$ for the staggered PRT scheme with one-overlay resolution. The value, $\kappa = 2/3$, seems to be optimum regardless of the algorithm for velocity retrieval; hence, it is exclusively used in this paper.

3. Spectral processing of staggered PRT sequence

Spectral processing of the staggered PRT sequence (Sachidananda et al. 1999; Sachidananda and Zrnic 2000) was proposed for filtering the ground clutter. In this procedure, the staggered sample sequence is first converted to a uniform sequence by inserting zeros in places where the samples are missing. This, of course, is possible if T_1 and T_2 are selected appropriately, as integer multiples of a basic uniform PRT, T_u. The constructed uniform sequence (with inserted zeros) is expressed as a product of a uniform sample sequence (without missing values) and a binary code sequence. Thus, the spectrum of the staggered PRT sequence is a convolution of the signal spectrum with the spectrum of the binary coded sequence. The spectral processing is proposed only if clutter needs to be filtered; otherwise, the pulse pair processing is sufficient to estimate the spectral moments and is better in terms of estimate errors and computational complexity. The idea central to the filtering of the clutter from the staggered PRT sequence is to recover the spectral coefficients of the weather signal in the region where the clutter and signal spectra overlap. Conventional filtering techniques cannot distinguish between the signal and the clutter power in a given spectral coefficient, and filtering one automatically eliminates the other. In this new procedure,



(a) Staggered PRT sampling scheme: one overlay, κ =2/3.



(b) Equivalent uniform PRT sampling for range at τ.

FIG. 1. The staggered PRT sampling scheme. Stagger ratio is 2/3, and echoes with one overlay are shown. Echoes from region 2 are clear of overlay; echoes from regions 1 and 3 overlay each other on alternate samples.

the modulation properties of the code sequence and the "narrow" spectra condition (i.e., the spread of the nonzero spectral coefficients is less than 1/5th of the $-v_a$ to $+v_a$ interval for $\kappa = 2/3$) allow us to completely filter the ground clutter while retaining some fraction of the signal power. An estimate of a complex clutter spectral coefficient is obtained by projecting the complex spectrum code vector (an appropriate column vector of the convolution matrix) onto the set of spectral coefficients where clutter is expected (five coefficients for $\kappa = 2/3$). This estimated clutter vector is subtracted from the set of coefficients. Because the different spectral code vectors are linearly independent, the signal power is not completely filtered except when the weather signal velocity is very close to zero. (Ground clutter has a narrow spectrum that is always centered on zero velocity.) This residual signal power is used to restore the signal to its original value with the help of some additional information. This is possible because the residual power left in the spectral coefficient after the clutter is filtered is a known fraction of the original signal power. It is also possible to restore the complex signal coefficient, not just the magnitude, but for spectral moment estimation, the power spectrum is sufficient (Sachidananda et al. 2001). The correction factor can be computed, provided the location of the original signal component can be determined. This is accomplished by obtaining an initial approximate velocity estimate with the partial signal power. This last step is termed the *bias removal* procedure; it eliminates the small bias in the velocity estimate by restoring the signal lost while filtering the clutter.

4. Unambiguous range extension by separating overlaid signals

In the staggered PRT sampling, one-overlay situation occurs if weather echoes extend to a maximum range, r_{a2} (see Fig. 1). The transmitted pulses are at intervals T_2 and T_1, alternately, and the return signal amplitude is depicted as a rugged graph. For the stagger ratio $\kappa = 2/3$, T_1 and T_2 are related to the basic time unit T_u by $T_1 = 2T_u$, and $T_2 = 3T_u$. Let the echoes extend over a delay time T_2 after the transmitted pulse, so that signal overlay occurs up to a delay time T_u after the T_2 pulse transmission, but there is a clear range of r_{a1} without overlaid echoes after the T_1 pulse transmission.

We divide the range r_{a2} into three regions corresponding to the delay time $\tau \leq T_u$, $T_u \leq \tau \leq 2T_u$, and $2T_u \leq \tau \leq 3T_u$. Echoes from region 1 and region 3 can be overlaid after the T_2 pulse transmission, whereas for the region 2, there is no overlay. There can also be ground clutter in the region 1, but it usually occurs in ranges less than about 20 km at most elevations.

In practice, several different combinations of strengths of the clutter, signal, and overlay powers are possible, and the spectral moment calculation has to be appropriately channeled in the staggered PRT algorithm. In this paper, we examine the one-overlay problem and develop a method for estimating the spectral moments of echo signals from both ranges involved.

a. The one-overlay resolution: Theory

For compactness, let *a* refer to echo (or signal) from the region 1 (0 to $r_{a1}/2$, or at a delay time τ; $\tau \leq T_u$), and let *b* refer to echo from the region 3 (r_{a1} to r_{a2}, or at a delay time $T_1 + \tau$). The sample spacing along a radial, $\delta\tau$, (range gate spacing in time) is chosen such that T_u and τ are integral multiples of $\delta\tau$. Therefore, the samples in sequences (as in Fig. 1) from which "*a*" or "*b*" signals will be retrieved are spaced at integer multiples of T_u.

We start the transmission sequence with T_2 first (i.e., $T_2\ T_1\ T_2\ T_1 \cdots$ Fig. 1); the reason for this choice will be clear later when we discuss the processing algorithm. The sample sequence with a delay time τ (in the long and short PRT) is denoted by $[v_1, v_2, v_3, v_4, \ldots$, etc.], and the samples at delay time $(T_1 + \tau)$, within the longer PRT, are represented by $[v_1', v_3', v_5', v_7', \ldots$, etc.], (Fig. 1). Note that this set has only odd-numbered samples available from the T_2 pulse transmission. The first sequence corresponds to the staggered PRT samples for estimating the signal *a*; in this sequence, the odd-numbered samples contain overlaid echo *b*. Similarly, we can form the staggered PRT sequence for signal *b* by replacing the even-numbered samples in the first set by the second set of samples, i.e., $[v_1, v_1', v_3, v_3', v_5, v_5', \ldots$, etc.] (Fig. 1). The odd-numbered unprimed samples are sums of *a* and *b* signals. Thus, $[v_1, v_2, v_3, v_4, \ldots$, etc.] is the sequence for estimating parameters of the *a* echo; the odd-numbered samples have overlaid signal *b*. The $[v_1, v_1', v_3, v_3', v_5, v_5', \ldots$, etc.] sequence is for estimating the *b* echo, and the odd-numbered unprimed samples contain overlaid signal *a*.

Now, to convert these nonuniform sequences into uniform sequences, we insert zeros in the place of missing samples. The time series sequence for extracting signal a is [v_1, 0, 0, v_2, 0, v_3, 0, 0, v_4, 0, . . . , etc.], and for extracting signal b, it is [v_1, 0, v_1', 0, 0, v_3, 0, v_3', 0, 0, v_5, 0, v_5', . . . , etc.]. The code sequence for retrieving parameters of a echo is [10010 . . .], and for the b echo, it is [10100 . . .]. Thus, the uniform sample sequence has a length $N = 5M/2$. Let us use the notation p_1, v_1, w_1 for the power, velocity, and spectrum width of the echo signal a; and p_2, v_2, and w_2 for the spectral moments of b echo. To explain the staggered PRT one-overlay resolution algorithm, assume that p_2 is larger than p_1. If the weaker signal velocity v_1 can be recovered, the stronger signal velocity is also recoverable; hence, we concentrate on the estimation of the weaker signal velocity. If the signal a is stronger, the same procedure applies, but the sequences must be interchanged.

The uniform sample sequence, $v = [v_i,\ i = 0, 1, 2, 3, \ldots, N - 1]$, (either for retrieving a or b echo) can be represented as a product of the corresponding code sequence $c = [c_i,\ i = 0, 1, 2, 3, \ldots, N - 1]$ and $e = [e_i,\ i = 0, 1, 2, 3, \ldots, N - 1]$, where e is the desired but unknown time series of the signal sampled at T_u intervals:

$$v_i = c_i e_i; \qquad i = 0, 1, 2, 3, \ldots, N - 1. \tag{3}$$

The number of uniform samples is N; $N = 5M/2$, where M is the number of staggered PRT samples. Therefore, the spectrum of v can be represented as a circular convolution (★) of the spectrum of the code c and the spectrum of the signal e,

$$\mathrm{DFT}\ (v) = \mathrm{DFT}\ (c) \bigstar \mathrm{DFT}\ (e), \tag{4}$$

where DFT represents the discrete Fourier transform of the quantity in parentheses. Equation (4) can be written in a matrix form as

$$\mathbf{V} = \mathbf{CE}, \tag{5}$$

where $\mathbf{V}$ and $\mathbf{E}$ are $(N \times 1)$ column vectors containing the DFT coefficients V_k and E_k of the corresponding time sequences, v and e; C is the convolution matrix (size: $N \times N$) whose column vectors are cyclically shifted versions of C_k. We follow the convention that lowercase letters represent the time domain quantities and the uppercase letters the corresponding Fourier domain quantities (i.e., DFT coefficients). Boldface sans serif letters are used for matrices. To preserve the power in the spectrum, the convolution matrix C is normalized such that each column vector is a unit vector (i.e., the norm of each column is unity). In each row, the convolution matrix has only five nonzero elements (or column) separated by $N/5$ coefficients, and hence, (5) can be rearranged such that C is a 5×5 matrix, and the other two matrices are $5 \times (M/2)$ matrices. The rearranged matrix equation is

$$\mathsf{V}_r = \mathsf{C}_r\, \mathsf{E}_r, \tag{6}$$

where C_r is obtained by deleting the rows and columns of C containing zero in the first column and the first row, respectively. Matrices V_r and E_r are obtained from $\mathbf{V}$ and $\mathbf{E}$ by arranging the elements row wise into $5 \times (M/2)$ size matrices.

To understand the overlay resolution procedure, first let us examine the spectrum of the code sequences, [10000 . . .], [00010 . . .], which are the two components of the staggered PRT code [10010 . . .] for retrieving signal a and the corresponding rearranged convolution matrices, C_{1r}, C_{2r}, such that $\mathsf{C}_{1r} + \mathsf{C}_{2r} = \mathsf{C}_r$. They are given by

$$\mathrm{abs}\{\mathsf{C}_r\} = \begin{bmatrix} 1.0 & 0.309 & 0.809 & 0.809 & 0.309 \\ 0.309 & 1.0 & 0.309 & 0.809 & 0.809 \\ 0.809 & 0.309 & 1.0 & 0.309 & 0.809 \\ 0.809 & 0.809 & 0.309 & 1.0 & 0.309 \\ 0.309 & 0.809 & 0.809 & 0.309 & 1.0 \end{bmatrix}, \tag{7}$$

$$\mathrm{phase}\{\mathsf{C}_r\} = \begin{bmatrix} 0^\circ & -144^\circ & -108^\circ & 108^\circ & 144^\circ \\ 144^\circ & 0^\circ & -144^\circ & -108^\circ & 108^\circ \\ 108^\circ & 144^\circ & 0^\circ & -144^\circ & -108^\circ \\ -108^\circ & 108^\circ & 144^\circ & 0^\circ & -144^\circ \\ -144^\circ & -108^\circ & 108^\circ & 144^\circ & 0^\circ \end{bmatrix}, \tag{8}$$

$$\mathrm{abs}\{\mathsf{C}_{1r}\} = \mathrm{abs}\{\mathsf{C}_{2r}\} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}, \tag{9}$$

$$\mathrm{phase}\{\mathsf{C}_{1r}\} = \begin{bmatrix} 0^\circ & 0^\circ & 0^\circ & 0^\circ & 0^\circ \\ 0^\circ & 0^\circ & 0^\circ & 0^\circ & 0^\circ \\ 0^\circ & 0^\circ & 0^\circ & 0^\circ & 0^\circ \\ 0^\circ & 0^\circ & 0^\circ & 0^\circ & 0^\circ \\ 0^\circ & 0^\circ & 0^\circ & 0^\circ & 0^\circ \end{bmatrix}, \tag{10}$$

$$\mathrm{phase}\{\mathsf{C}_{2r}\} = \begin{bmatrix} 0^\circ & 144^\circ & -72^\circ & 72^\circ & 144^\circ \\ -144^\circ & 0^\circ & 144^\circ & -72^\circ & 72^\circ \\ 72^\circ & -144^\circ & 0^\circ & 144^\circ & -72^\circ \\ -72^\circ & 72^\circ & -144^\circ & 0^\circ & 144^\circ \\ 144^\circ & -72^\circ & 72^\circ & -144^\circ & 0^\circ \end{bmatrix}. \tag{11}$$

The coefficients of C_r in the above expressions are normalized so that the highest coefficient is unity. This is to show the relative amplitudes of the coefficients. In the actual algorithm, the normalization is with respect to the power such that the sum of the magnitude square of each column elements (or row) is made unity. It is particularly important to note that all the elements of C_{1r} are equal, and the elements of C_{2r} differ only in the phase; all magnitudes are the same. Further, the mean value of each column of C_{2r} is identically zero; thus, the mean value of each column of C_r and C_{1r} is the same and equals to 0.5, or half the maximum coefficient of $\mathbf{C}_r$. These properties are used in the overlay resolution algorithm.

We start with the assumption that the signal spectrum is "narrow"; i.e., the spread of the nonzero spectral coefficients of the signal is limited to $2v_a/5$. Now, if we take the DFT of the full time series of the signal (samples at intervals T_u with no missing samples and no overlay) and row-wise rearrange the coefficients into ($5 \times M/2$) size matrix, then each column will have only one nonzero coefficient. A convolution of this spectrum with any one of the code matrices ($\mathbf{C}_r$, $\mathbf{C}_{1r}$, or $\mathbf{C}_{2r}$) will spread the power only among the coefficients of the same column. Thus, (6) is a set of $M/2$ independent equations, one for each column of $\mathbf{E}_r$ and $\mathbf{V}_r$. Therefore, to understand the decoding scheme, it is sufficient to consider one such matrix equation.

Let all the spectra be represented as row-wise rearranged matrices of ($5 \times M/2$) size; that is, take the DFT of the time series sequence, and rearrange the spectral coefficients row-wise to form the matrix of complex coefficients. The rearranged matrices are designated by the subscript *r*. Let the signal *a* (samples at T_u interval and with no overlay) spectrum be $\mathbf{S}_{1r}$ and, similarly, the spectrum of the signal *b* be $\mathbf{S}_{2r}$. In terms of these two spectra, the staggered PRT signal for retrieving the *a* echo (with the *b* echo overlay in one-half of the samples) can be represented as a sum of convolutions of these two spectra with the appropriate convolution matrices. The spectrum, $\mathbf{E}_{1r}$, of the time series, $e_{s1} = [v_1, 0, 0, v_2, 0, \ldots]$, which has the signal *a* plus the signal *b* overlaid in odd-numbered samples, is written as

$$\mathbf{E}_{1r} = \mathbf{C}_r\mathbf{S}_{1r} + \mathbf{C}_{1r}\mathbf{S}_{2r}. \tag{12}$$

This is a set of $M/2$ equations, one for each column of the matrix, $\mathbf{E}_{1r}$ with the corresponding columns of $\mathbf{S}_{1r}$ and $\mathbf{S}_{2r}$ on the right-hand side. To examine one such equation let $\mathbf{e}_k = [\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3, \mathbf{E}_4, \mathbf{E}_5]^{\mathrm{T}}$ be one column (say *k*th column) of $\mathbf{E}_{1r}$; further, let $\mathbf{a}_k = [0, 0, \mathrm{a}_1, 0, 0]^{\mathrm{T}}$ be the *k*th column of $\mathbf{S}_{1r}$, and $\mathbf{b}_k = [0, 0, 0, \mathrm{b}_2, 0]^{\mathrm{T}}$ be the *k*th column of $\mathbf{S}_{2r}$, and in both, only one element is nonzero by virtue of the "narrow" spectra assumption. (The selection of a particular element as nonzero is arbitrary in this example.) It may be noted that the narrow spectra assumption is not necessary for the overlay resolution algorithm to work. It is an assumption needed for the magnitude deconvolution procedure for reconstructing the spectrum. The overlaid signal *b* is removed irrespective of its spectrum width, but the assumption is necessary because we need to recover the velocity at both ranges, and the magnitude deconvolution may be applied to the signal *b* also to recover its spectral moments. The spectrum width of the overlay signal also plays an important role in the recovery of the velocity of signal *a* because the decorrelation of the samples with time is a function of the spectrum width. The matrix equation is

$$\begin{aligned}\mathbf{e}_k &= \mathbf{C}_r\mathbf{a}_k + \mathbf{C}_{1r}\mathbf{b}_k, = [\mathbf{C}_{1r} + \mathbf{C}_{2r}]\mathbf{a}_k + \mathbf{C}_{1r}\mathbf{b}_k,\\ &= \mathbf{C}_{1r}[\mathbf{a}_k + \mathbf{b}_k] + \mathbf{C}_{2r}\mathbf{a}_k.\end{aligned} \tag{13}$$

Now, because $\mathbf{C}_{1r}$ is a matrix with equal elements, the contribution of the overlay in each of the elements of the column on the left-hand side is a constant (complex), and also the contribution of the odd-numbered samples of the sequence for retrieving the signal *a* is a constant. Therefore, if we subtract the mean value of the column from each element, we effectively delete all the contribution of the signal *b*. This is the reason for selecting the time series beginning with T_2 pulse. If not, the second convolution matrix would be $\mathbf{C}_{2r}$ corresponding to the code [00100, . . .], which will not produce a constant contribution. This looks very simple, but the recovery of the signal mean velocity is more subtle because we need the ($\mathbf{C}_{1r}\mathbf{a}_k$) part (all elements of the column are the same) of the sequence for retrieving the velocity from the signal *a*.

From the properties of the convolution matrices involved, it can be inferred that the constant ($\mathbf{C}_{1r}\mathbf{a}_k$) is equal (in amplitude and phase) to one of the coefficients of the column ($\mathbf{C}_{2r}\mathbf{a}_k$), which is the original signal component (i.e., the third element that we started with in the example). Therefore, the whole problem is reduced to determining which is the position of the original signal component in ($\mathbf{C}_{2r}\mathbf{a}_k$). In essence, one-half of the samples has no overlay, and the signal of this set with inserted zeros produces a spectrum which has five identical replicas of the original spectrum with relative phase shifts corresponding to the appropriate code. By choosing to start the sequence from the T_2 pulse transmission, we make the contribution of the other half that is buried in overlay to be an identical spectrum (five replicas) but with no relative phase shifts between the five components. Of the five replicas in the first set, only one has the same phase as the second set, and we need to identify it to resolve the ambiguity in the velocity.

To determine the position of the original signal component in the columns of ($\mathbf{C}_{2r}\mathbf{a}_k$), we go through a series of steps. Each element of the column $\mathbf{C}_{1r}[\mathbf{a}_k + \mathbf{b}_k]$ has two parts, the signal *a* and the *b* overlay, and it is not possible to split it into two parts without additional information. This information is derived from the other set of $M/2$ samples which contain only the overlaid signal. Take the set of samples, $e_2 = [0, 0, v_1', 0, 0, 0, 0, v_3', 0, 0, \ldots, \text{etc.}]$, and compute the DFT to get the rearranged matrix, $\mathbf{E}_{2r}$. The magnitude of the elements of the *k*th column of this matrix (all magnitudes are the same) gives an estimate of the magnitude of the overlay component in $\mathbf{C}_{1r}[\mathbf{a}_k + \mathbf{b}_k]$. If we compute $\{\mathbf{C}_{1r}[\mathbf{a}_k + \mathbf{b}_k] - \mathbf{C}_{2r}\mathbf{a}_k\}$, one of the elements of this column vector will have the same magnitude as the elements of the *k*th column of $\mathbf{E}_{2r}$. Therefore, the column matrix $\mathbf{g}$, given by the operation

$$\mathbf{g} = \mathrm{abs}[|\mathbf{C}_{1r}[\mathbf{a}_k + \mathbf{b}_k] - \mathbf{C}_{2r}\mathbf{a}_k| - |\mathbf{E}_{2r}(k)|], \tag{14}$$

will have five values which are the differences between the five possible vectors and the estimate of $\mathbf{E}_{2r}(k)$. The position of the element with the smallest magnitude is the most likely candidate for the correct position of the original signal component in $(\mathbf{C}_{2r}\mathbf{a}_k)$. This procedure is applied to all the columns to obtain an index array, $\mathbf{n}_k$, of $M/2$ elements (row matrix) containing the most likely row index of the original signal component in each column of $(\mathbf{C}_{2r}\mathbf{S}_{1r})$. Once the position of the element is known, we can reconstruct the signal a by adding that component to the already available other half of the spectrum. The reconstructed spectrum of the signal a alone is

$$\mathbf{E}_{tr} = \mathbf{C}_r\mathbf{S}_{1r}. \quad (15)$$

Because the convolution matrix, $\mathbf{C}_r$, is singular, we recover the magnitude spectrum, abs$(\mathbf{S}_{1r})$, using the magnitude deconvolution procedure; that is,

$$\text{abs}(\mathbf{S}_{1r}) = \{\text{abs}(\mathbf{C}_r)\}^{-1}\,\text{abs}(\mathbf{E}_{tr}). \quad (16)$$

Generally, (16) does not hold, but it can be shown that it is valid for the narrow spectra (Sachidananda and Zrnic 2000). It may be noted that abs$(\mathbf{C}_r)$ is nonsingular, and the power spectrum is sufficient to estimate the autocorrelation; the phases are not required.

The above procedure essentially gives the most probable position of the correct element, but it can fail due to several reasons. The most important one is the correlation between the estimate of the overlaid signal component and the actual overlaid signal component. This correlation is a function of the spectrum width of the overlay signal; if the width increases, the correlation coefficient becomes smaller. The other parameter that affects the velocity recovery is the overlay power ratio, p_2/p_1. The criterion for success is that the uncertainty in the estimated magnitude of $\mathbf{C}_{1r}\mathbf{b}_k$ must be less than the magnitude of the spectral coefficients of $\mathbf{C}_{2r}\mathbf{a}_k$. This is roughly related to the overlay power ratio but not necessarily for all the columns of the $\mathbf{E}_{1r}$. The p_2/p_1 ratio is the overall power ratio for the whole spectrum, but the overlay power ratio for each column can be completely different. The overlay power ratio for each column cannot be easily computed because we do not know the exact overlay power in each spectral coefficient.

Note that the spectral coefficients $\mathbf{E}_{2r}$ are not obtained from the same set of samples as that present in the overlaid samples, but from a sample set that is delayed by T_1. The correlation between the overlaid samples and this set is crucial for overlay resolution; that is, the accuracy of our estimate of overlay component from the nonoverlaid sample set for the same range depends on the autocorrelation at lag T_1, and the spectrum width is a measure of the autocorrelation. Thus, if the overlaid signal has a narrow spectrum, the recovery is very good. This is an important point to note for the case of overlay resolution in the presence of the ground clutter. We shall discuss this topic further in section 5c. If we take the spectrum of the alternate set of staggered PRT samples of a weather echo (i.e., sequences $\{v_1, 0, 0, 0, 0, v_3, 0, 0, 0, \ldots, \text{etc.}\}$ and $\{0, 0, 0, v_2, 0, 0, 0, 0, v_4, \ldots, \text{etc.}\}$), and define a differential power ratio R_d as the ratio of total power in the spectrum to the power in the difference spectrum (defined as the coefficient wise difference in the spectral magnitudes), we get an idea of the overlay power ratios for which velocity can be resolved. But we cannot quantify overlay resolution from R_d. The larger the ratio, the better the overlay resolution. In Fig. 2 is a plot of R_d as a function of the spectrum width of the overlaid signal for different M and delay times, T_1 and T_2, generated using a simulated time series. We encounter both delays, T_1 and T_2, depending on whether the signal a or signal b is overlaid. It is seen that for a larger number of samples, R_d is higher for narrow spectra, but for $w > 4$ m s^{-1}, the difference is marginal. Because R_d decreases with spectrum width, overlay cancellation is better if the overlay signal has narrower spectra.

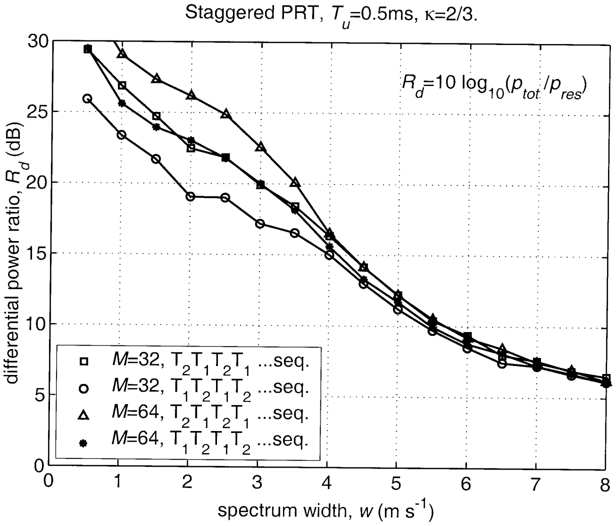


FIG. 2. The differential power ratio as a function of the spectrum width of the signal.

Simulation study has shown that for $w < 3$ m s^{-1} the overlay resolution is very good, but for larger widths the failure rate increases. (Quantitative results are in section 5.) The proposed overlay resolution algorithm uses some of the known properties of the spectrum to further improve the velocity recovery. Without these, the velocity recovery fails for overlay ratios larger than about 10 dB.

A typical echo signal with Gaussian narrow spectrum, when modulated by the staggered PRT code (10100 . . .), has five replicas separated by 1/5th of the $\pm v_a$ interval. For such a spectrum, we can determine the shape of the power spectrum in the 1/5th of $\pm v_a$ interval from one-half of the samples that have no overlay. From this shape, we estimate an approximate position of the minimum of the spectral power envelope function. Now, we know that the position of the spectrum replica is between two consecutive minima that are exactly $N/5$ (or $M/2$) coefficients apart, and there are five such segments in the entire spectrum. One of these segments is at the position of the original spectrum, and the other four are the replicas generated by the convolution with the code spectrum. The DFT coefficients are cyclic; hence, if the index exceeds N in the last segment, it is to be joined with the beginning of the spectrum. Similarly, the rearranged matrix has five rows of $N/5$ coefficients, with row indices 1 to 5. Therefore, for any segment, the row index can have at best two indices, and these two indices will differ by one. Further, the change over from one index to the next occurs at the minimum of the spectral power envelope function. Hence, for the original signal, the correct index array, $\mathbf{n}_k$, can have only two segments, with index values differing by one in the descending order (cyclic over 5 to 1). With this known property, we can correct many of the missed indices in $\mathbf{n}_k$.

First, we determine the position of the minimum of the spectrum envelope. To do this, we smooth the envelope of the first 1/5th of the spectrum ($M/2$ coefficients of $\mathbf{C}_{2r}\mathbf{S}_{1r}$, or the first row) into six to eight segments, and average the powers in these segments. We find the segment that has lowest power, and position the minimum at the center of this segment. This, of course, is an approximate position, but it is not critical because it is at the power minimum. Once the position of the minimum is obtained, we can cut the index array $\mathbf{n}_k$ into two pieces and select the piece that has a larger number of elements in it. Now, determine the index that has appeared the maximum number of times in this segment of $\mathbf{n}_k$ and assign this index to all the elements of the segment as the most likely candidate. The index value for the other segment of $\mathbf{n}_k$ is determined as one less or one more, depending on whether it is on the right or on the left of the previous segment. The indices are in the descending order and cyclic between 5 and 1. This modified index array, $\mathbf{n}_k$, when used in the reconstruction of the spectrum of the signal a, gives a much better estimate of the weaker signal velocity.

The overlay resolution procedure explained in this section is applied to the weaker of the two signals involved. There is a significant amount of computation involving fast Fourier transform (FFT); hence, it is important to minimize the use of the overlay algorithm. For the stronger signal, we can generally apply the pulse-pair algorithm to estimate the velocity without removing the overlaid echo. The overlay power marginally increases the error in the velocity estimate. The staggered PRT pulse-pair algorithm uses the phases of autocorrelations, R_1 and R_2, to estimate two aliased velocities, v_1 and v_2, within the intervals, $\pm v_{a1}$ and $\pm v_{a2}$, respectively. Next, we compute all possible aliases for the two cases within the $\pm v_a$ interval (v_{a1}, v_{a2}, and v_a are the unambiguous velocities corresponding to the uniform PRTs, T_1, T_2, and T_u, respectively). Of these two sets, only one value is common, and that is the correct velocity. The mean power is estimated from the $M/2$ samples which are free of overlay for each of the ranges.

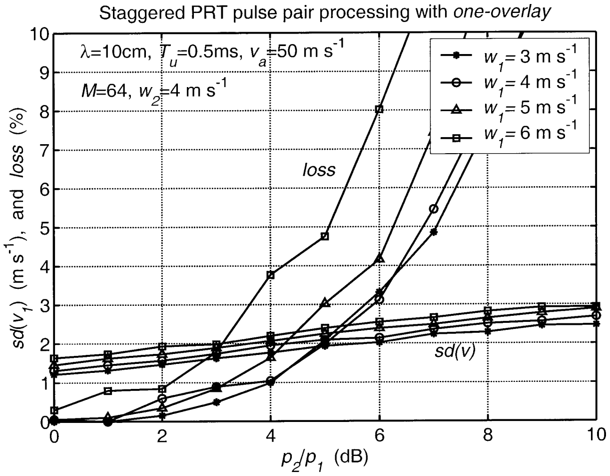


FIG. 3. Two performance metrics of the staggered PRT pulse-pair processing in the presence of one overlay: the std dev of velocity estimates of the weaker signal and percent of lost estimates.

We have examined the effectiveness of the pulse-pair algorithm for the weaker signal using simulated time series and estimated the maximum allowable overlay power ratio for which the pulse-pair algorithm can still function. The result from a large number of simulations is presented in Fig. 3. Each point (represented by a symbol) on the plot is obtained from 2020 simulations with velocities spread over a $\pm 0.95 v_a$ interval. The last 5% is excluded because the velocity folding at the extreme ends does not represent the failure of the algorithm. Percentage *loss* is defined as the ratio of the number of times the velocity dealiasing failed to the total number of simulations. Note that the *loss* is independent of the spectrum width of the overlaid signal. It is a function of overlay power (Fig. 3). The *loss* increases with the signal a spectrum width w_1. The standard error, sd(v), in the velocity estimate is calculated from the correctly dealiased velocities to avoid biases due to outliers; whenever the dealiasing fails, the velocity is off by v_{a1}, and the sd(v) increases abruptly, which is unrepresentative of the true standard error. It is seen from Fig. 3 that if we allow a *loss* of about 2%, we could use the pulse-pair algorithm for both the signals (a and b) for p_2/p_1 up to 3 dB. If a *loss* of 10% could be tolerated, we could use it up to 6 dB.

b. The one-overlay resolution algorithm

The steps involved in estimating the spectral moments from the staggered PRT time series with one overlay are listed next. Explanations are given in curly brackets, { }. Lowercase letters denote time domain quantities, and capital letters denote spectral domain quantities.

Bold sans serif fonts, both upper- and lowercase, represent matrices.

1) Input the staggered PRT sample sequence, $[v_1, v_1', v_2, v_3, v_3', v_4, v_5, v_5', \ldots]$.
   The sequence starts with T_2 pulse transmission, the number of transmitted pulses is M, and the total number of samples in the sequence is $3M/2$. See Fig. 1 for timing notation.
2) Compute the mean power estimates, $p_1$and p_2, from the samples,

$$p_1 = \frac{2}{M} \sum_{i=2,4,\ldots}^{M} |v_i|^2, \quad \text{and} \quad p_2 = \frac{2}{M} \sum_{i=1,3,5,\ldots}^{M} |v_i'|^2.$$

   Use the ratio p_2/p_1 to decide the path of computation. If $|p_2/p_1| < 6$ dB, use the staggered PRT pulse-pair algorithm for parameters of both signal a and signal b. For $|p_2/p_1| > 6$ dB, the overlay separation is applied only for recovering the weaker signal parameters, and the stronger signal parameters are estimated using the pulse-pair algorithm. The steps below (3–15) are for recovering the weaker signal velocity using the overlay resolution algorithm, $p_1 < p_2$.
3) Form four uniformly spaced sample sequences of length $N = 5M/2$ by inserting zeros as follows:

$$e_{s1} = [v_1, 0, 0, v_2, 0, v_3, 0, 0, v_4, \ldots];$$

   signal a + signal b overlay in v_1, v_3, etc.;

$$e_{s2} = [v_1, 0, v_1', 0, 0, v_3, 0, v_3', 0, 0, \ldots];$$

   signal b + signal a overlay in v_1, v_3, etc.;

$$e_1 = [0, 0, 0, v_2, 0, 0, 0, 0, v_4, 0, \ldots];$$

   signal a only; and

$$e_2 = [0, 0, v_1', 0, 0, 0, 0, v_3', 0, 0, \ldots];$$

   signal b only.
4) Compute the DFT of sequences e_{s1} and e_2 after multiplying by von Hann window weights (Doviak and Zrnic 1993), h_n:

$$E_1 = \mathrm{DFT}(e_{s1}h_n) \quad \text{and} \quad E_2 = \mathrm{DFT}(e_2 h_n).$$

5) Rearrange the coefficients of E_1 and E_2 into $5 \times M/2$ matrices:

$$E_1 \rightarrow \mathbf{E}_{1r} \text{ and } E_2 \rightarrow \mathbf{E}_{2r}.$$

   Rearranged matrix is explained in section 4a; $k = 2/3$ is assumed.
6) Compute mean value of each column of $\mathbf{E}_{1r}$, and form a matrix, $\mathbf{E}_m$, of the same size as $\mathbf{E}_{1r}$ with elements of each column as the mean value of that column:

$$\mathbf{E}_m = \begin{bmatrix} \overline{E_m(1)} & \overline{E_m(2)} & \cdots & \overline{E_m\left(\frac{M}{2}\right)} \\ \overline{E_m(1)} & \overline{E_m(2)} & \cdots & \cdot \\ \overline{E_m(1)} & \overline{E_m(2)} & \cdots & \cdot \\ \overline{E_m(1)} & \overline{E_m(2)} & \cdots & \cdot \\ \overline{E_m(1)} & \overline{E_m(2)} & \cdots & \overline{E_m\left(\frac{M}{2}\right)} \end{bmatrix}.$$

   where

$$\overline{E_m(k)} = \frac{1}{5} \sum_{i=1}^{5} E_{1r}(i, k); \qquad k = 1, 2, 3, \ldots, \frac{M}{2}.$$

7) Compute the following matrices, $\mathbf{G}$ and $\mathbf{H}$:

$$\mathbf{G} = \mathrm{abs}[|2\mathbf{E}_m - \mathbf{E}_{1r}| - |\mathbf{E}_{2r}|] \quad \text{and}$$

$$\mathbf{H} = [\mathbf{E}_{1r} - \mathbf{E}_m].$$

8) Find the row index of the smallest element in each column of $\mathbf{G}$ to form an integer row vector $\mathbf{n}_k$ of $M/2$ elements. Note that $\mathbf{n}_k$ element values are any one of the row indices, 1–5.
9) Compute the row vector, $\mathbf{h}_1$, of the spectral power coefficients from the first row of $\mathbf{H} = [\mathbf{E}_{1r} - \mathbf{E}_m]$,

$$\mathbf{h}_1 = |\text{1st row of } [\mathbf{E}_{1r} - \mathbf{E}_m]|^2.$$

   All rows of $\mathbf{H}$ give the same result because the coefficients have equal magnitudes in all rows.
10) Segment the row vector $\mathbf{h}_1$ into 6–10 equal length segments, depending on the number of elements ($M/2$ must be divisible by the number of segments to obtain equal length segments), and determine the segment which has the lowest power. Determine the index of the coefficient at the middle of this segment. Let k be this index. Note that $\mathbf{h}_1$ and $\mathbf{n}_k$ have the same length, $M/2$. The number of segments can be selected conveniently based on the number of samples. The indicated value, between 6 and 10, is not very critical. For $M = 64$, 8 is found to be a good choice.
11) Divide $\mathbf{n}_k$ into two segments: the first segment, elements 1 to k, and the second segment, elements $k + 1$ to $M/2$. Two cases to be considered are $k < M/4$, and $k < M/4$.
    If $k > M/4$, then find the index x that appears the maximum number of times in the first segment of $\mathbf{n}_k$ and replace all the elements of the first segment of $\mathbf{n}_k$ with x. The elements of the first segment of $\mathbf{n}_k$ are replaced by $(x - 1)$. If $(x - 1)$ is zero, the value is set to 5. (Indices are cyclic, 5 to 1, in descending order.)
    If $k < M/4$, then find the index, x, that appears the maximum number of times in the second segment of $\mathbf{n}_k$, and replace all the elements of the second segment of $\mathbf{n}_k$ with x. The elements of the first

segment of $\mathbf{n}_k$ are replaced by $(x + 1)$. If $(x + 1)$ is 6, the value is set to 1.

12) Now, select the $[\mathbf{n}_k(k)]$th element of the kth column of $\mathbf{H}$, $k = 1, 2, \ldots, M/2$, to form a row vector of $M/2$ elements, and repeat these rows 5 times to form the complete $5 \times M/2$ size correction matrix, $\mathbf{E}_{1c}$. The recovered spectrum of the weaker signal $\mathbf{E}_t$ is obtained by adding the correction matrix to $\mathbf{H}$:

$$\mathbf{E}_t = [\mathbf{H} + \mathbf{E}_{1c}] = [\mathbf{E}_{1r} - \mathbf{E}_m + \mathbf{E}_{1c}].$$

Note that the matrix $\mathbf{H}$ corresponds to the spectral components of sequence e_1, and $\mathbf{E}_{1c}$ is the recovered spectrum of the sequence $[v_1, 0, 0, 0, 0, v_2, 0, 0, 0, 0, \ldots, \text{etc.}]$ with the signal b overlay part removed from these samples. The spectrum $\mathbf{H}$ is computed the way it is indicated in step 7 because it involves one DFT operation less.

13) Apply the magnitude deconvolution to the matrix, $\mathbf{E}_t$, to reconstruct the complete spectrum of the weaker signal $\mathbf{S}_{1r}$:

$$\text{abs}\{\mathbf{S}_{1r}\} = [\text{abs}\{\mathbf{C}_r\}]^{-1}\,\text{abs}\{\mathbf{E}_t\},$$

and rearrange abs $\{\mathbf{S}_{1r}\}$ into a column matrix $\mathbf{S}_1$.

14) Compute the autocorrelation $R(T_u)$ using the elements of $\mathbf{S}_1$ and the mean velocity from the phase of $R(T_u)$:

$$R(T_u) = \frac{1}{N}\sum_{k=0}^{N-1} |S_1(k)|^2 e^{j2\pi k/N},$$

$$v_1 = (v_a/\pi)\arg\{R(T_u)\}; \qquad v_a = \lambda/(4T_u).$$

15) The spectrum width is computed using Eq. (6.27) of Doviak and Zrnic (1993):

$$w_1 = (\lambda/2\pi T\sqrt{2})|\ln[p/|R(T_u)|]|^{1/2}$$
$$\text{sgn}[\ln(p/|R(T_u)|)].$$

Here, the mean power p is computed from the reconstructed spectrum and not the estimate obtained earlier in step 3.

16) For signal b mean velocity v_2 and spectrum width w_2, use the pulse pair algorithm.

Note: If $p_2/p_1 < -6$ dB, follow the procedure in steps 3–15, with sequences for retrieving signal a and signal b interchanged. The code matrices are different for these two sequences; however, the magnitude deconvolution matrix is the same for both.

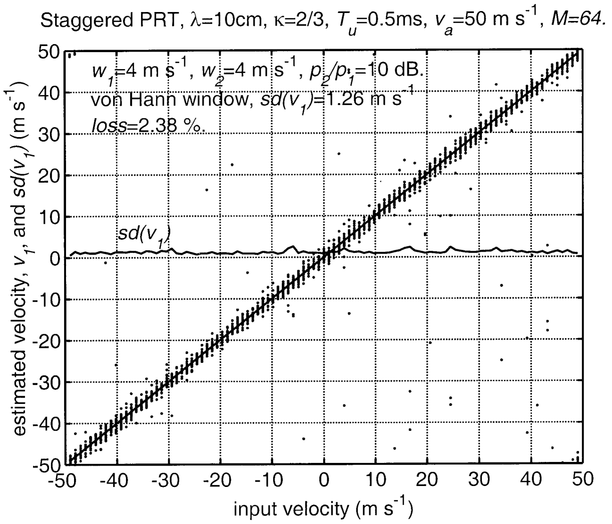


FIG. 4. The estimated velocity of the weaker signal obtained from simulated time series with one overlay, for $w_1 = 4$ m s^{-1}, $w_2 = 4$ m s^{-1}, and $p_2/p_1 = 10$ dB. The simulation parameters are written in the figure. Each point is one simulation, and there are 20 simulation points at each of the 101 velocities. The sd(v) is calculated from the correctly dealiased values only; the outliers are discarded. The mean of the sd(v) and the percent loss are also indicated.

5. Performance and practical aspects

a. Simulation results

To evaluate the performance of the overlay resolution algorithm given in section 4b, a large number of simulations were carried out with different input parameters. There are a number of variables to be considered, such as the overlay power ratio p_2/p_1; spectral moments of the two signals involved, PRTs, SNR; number of samples; etc. We selected 64 staggered PRT samples, a large SNR (>30 dB) for both signals, a typical weather radar wavelength of 10 cm, and $T_u = 0.5$ ms with a stagger ratio $k = 2/3$ (which gives an unambiguous velocity, $v_a = 50$ m s^{-1} and $r_{a2} = 225$ km) for our simulation results presented in this section. The most critical and difficult parameter to estimate is the weaker signal velocity, and this parameter is used to determine performance of the algorithm. In general, if the weaker signal velocity can be recovered, all other parameters are also recoverable. First, we present some sample scatterplots of the estimated velocity, v_1, of the weaker signal versus the velocity input to the simulation program to illustrate the quality of the estimates. Figure 4 is a typical scatterplot of the estimated velocity versus the velocity input to the simulation program, with an overlay power ratio of 10 dB. The simulation parameters are indicated in the figure. The parameter loss is the percentage of the number of times the velocity recovery failed in 2020 simulations, with input velocities uniformly distributed over $\pm v_a$. The standard error, sd(v), is computed leaving out the missed velocity points or the outliers. The mean value of the sd(v) is 1.26 m s^{-1}, and the loss = 2.38% for this case. There are some outliers at the extreme edge of the velocity interval, $\pm v_a$, which are due to the aliasing and not due to the failure of the algorithm. The rest of the outliers (a region of velocity error, $|v_1 - v_{\text{in}}| > v_a/5$) appear in bands because of the mode of failure of the algorithm; it is not able to resolve the ambiguity. This is more apparent for narrower spectrum widths as shown in Fig. 5, for a different set of parameters. The overlay power ratio is

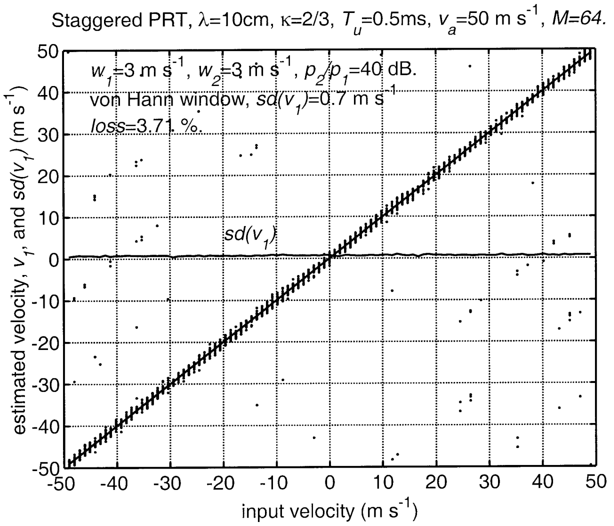


FIG. 5. The estimated velocity of the weaker signal obtained from simulated time series with one overlay, for $w_1 = 3$ m s^{-1}, $w_2 = 3$ m s^{-1}, and $p_2/p_1 = 40$ dB.

40 dB for this case and is almost at the limit of the overlay resolution with loss = 3.71%. If the spectrum width of the overlay is narrow, but the signal spectrum width is broad, then the standard error in the velocity estimate increases, which is because of the width itself, but the velocity recovery is very good. Figure 6 shows the loss as a function of the overlay power ratio with spectrum width of the overlay signal as a parameter. It can be seen that the loss is a strong function of w_2, and for widths greater than about 6 m s^{-1}, the loss is large even for $p_2/p_1 > 5$ dB for this particular set of parameters. It is possible to extend these results to other PRTs by normalizing the width with respect to the Nyquist velocity, v_a. Thus, if w_2 is less than $0.1v_a$, we can recover the velocity for $p_2/p_1 < 9$ dB (assuming that the allowable $loss = 10\%$) for any T_u. It is obvious that the shorter the T_u, the better will be the overlay resolution performance.

b. Constraints caused by weather signals

Three characteristics of weather echoes challenge the performance of the proposed one-overlay resolution algorithm. These are the large extent of the storms (beyond r_{a2}), large spectrum width, and large overlay power ratio. Each of these is briefly discussed in the context of possible implementation on the WSR-88D.

We argue that designs for mitigating range and velocity ambiguities must meet or exceed the current capabilities of the network. Therefore, the following features of the current volume coverage patterns should be preserved. At the two lowest elevations (0.5° and 1.5°), a long PRT, corresponding to unambiguous range of at least 460 km, has to be used for reflectivity estimation. These scans are to be complemented with scans with specially designed signal sequences for measuring the Doppler moments. Ground clutter canceling requirements dictate that a uniform PRT be used. Thus, the staggered PRT is disqualified at the two lowest elevations, where phase coding techniques are preferred. But the question of what to do at higher elevations, where the requirement for canceling ground clutter is relaxed, remains open. A staggered PRT with one-overlay resolution is a viable candidate, as explained next.

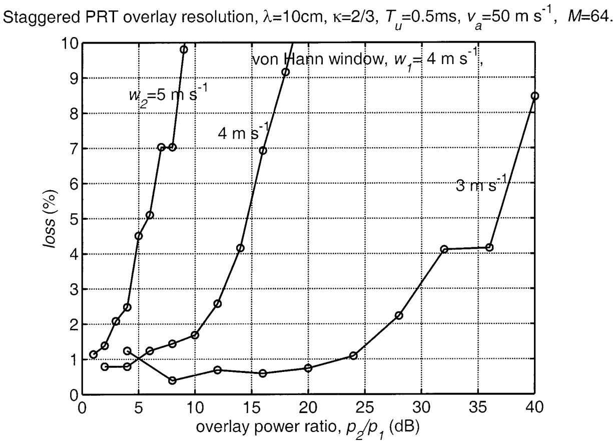


FIG. 6. The loss as a function of overlay power ratio p_2/p_1 with the spectrum width of the overlaid signal-*a* parameter.

Consider the scan at elevation of 2.5°. Useful r_{a1} could be between 150 and 200 km, and with the stagger ratio of 2/3, the corresponding r_{a2} is between 225 and 300 km. Take the more favorable (with respect to range ambiguities) value of $r_{a1} = 200$ km. The one-overlay could come from ranges between 200 and 300 km. But the height of a storm at 300 km would have to be over 18 km to be detected, and even if that were to happen, it would be over a miniscule area. Clearly, the one-overlay resolution would be applicable, and the condition that there is no overlay beyond $r_{a1}/2$ (imposed herein) holds. Nonetheless, with this large r_{a1}, the relative spectrum widths would be higher than if the r_{a1} of 150 km were chosen, and could cause degradation of velocity estimates. With this shorter r_{a1}, there would sometimes be overlaid echoes beyond 225 km, in which case the overlay resolution would fail and such data would have to be ignored. Note that the height of storms from which overlaid echoes (at the 2.5° elevation) could originate would have to be over 13 km. Few cells are that high, and if so, they usually cover a very small area.

Next, we examine the limitation imposed by the spectrum width. Recently, Fang and Doviak (2001) examined data obtained from several WSR-88D radars in a variety of weather events and obtained cumulative probabilities and median spectrum widths. Their list of events consists of 1) one clear-air event, 2) three stratiform rains, 3) three snow storms, 4) one widespread shower, 5) three isolated storms with tornadoes, 5) one multicell severe storm, and 6) three squall lines. In the first five event types, the median spectrum widths range from 1.4 to 2.3 m s^{-1}. The largest value was in one stratiform rain event in which percentage of widths greater than 4 m s^{-1} was 10%. Assuming that one-overlay situation occurs, a worst-case scenario is that all velocities in regions of $w > 4$ m s^{-1} are lost. The actual loss depends on p_2/p_1 (Fig. 3) and would be less than 10%. For example, if p_2/p_1 were 6 dB or less, and there were no widths larger than 6 m s^{-1}, the total percent loss of velocity data would be smaller than 0.08×0.1 (see Fig. 3), which amounts to 0.8%. But in this particular example, the height of precipitation was 7 km (above ground), and at 1.5° elevation, one overlay would occur. (The beam would exceed storm top at 180 km.) At 2.5° elevation scan, there would be no overlaid echoes.

The loss in the squall lines and multicell storms could be significant. The worst case analyzed by Fang and Doviak (2001) had a whopping 40% of spectrum widths larger than 6 m s^{-1}. The height of cells was 10.3 km. Therefore, if r_{a1} were chosen to be 150 km, there would be one-overlay situations in which many velocities could not be recovered. But, if r_{a1} were larger than 200 km, there would be no overlaid echoes whatsoever (at the 2.5° elevation), but errors in estimates due to relatively larger spectrum widths would increase.

Figure 3 suggests that to estimate both velocities, the two overlaid powers should be within 6 dB of each other. Although this excludes much of the data, it is better than the current procedure (Doviak and Zrnic 1993) which eliminates data if the overlaid echo powers are within 5 dB. Further, the unambiguous range for the staggered scheme is larger than in a uniform PRT scheme with the same unambiguous velocity. Therefore, the overlay would occur less often. Experience with real data will determine the value of the scheme, and it will tell if adaptive change of r_{a1} and r_{a2} radial by radial, could be made so that a best overall compromise is reached.

c. *Overlay and the ground clutter*

In the time series sample sequences for ranges in region 3, one-half of the samples are from region 1, which can have ground clutter along with the overlaid signal *a*. Because the ground clutter is confined to the ranges in region 1, it can be present in all the signal samples for the signal-*a* sequence but in only half the number of samples for retrieving signal-*b* sequence. The mean power estimate, p_1, (computed in step 2 of the algorithm, section 4b) will have ground clutter power added to the signal-*a* power, whereas p_2 will not be contaminated by the ground clutter power. As mentioned earlier in section 4a, if the spectrum width of the overlay signal is narrow ($w < 3$ m s^{-1}), it can be effectively removed, and the weaker signal velocity can be estimated accurately. Because the ground clutter spectrum width is very narrow ($w_c < 0.5$ m s^{-1}), the v_2, and w_2 can be estimated using the overlay resolution algorithm (steps 3–15). For this very narrow ground clutter width, v_2 can be recovered for overlay power ratio (or it is the same as clutter-to-signal ratio (CSR) for signal *b*, if signal *a* is absent) in excess of 50 dB. The ground clutter and signal *a* are the overlaid signals as far as signal *b* is concerned; thus, both are filtered simultaneously by the overlay resolution algorithm. If signal *a* is stronger than signal *b*, then its spectrum width, rather than the CSR will limit the v_2 recovery. The clutter filtering algorithm needs to be applied to recover parameters of the signal *a* only.

There are situations that need to be considered depending on the relative powers of the *a* and *b* signals and the ground clutter. The first problem we face is finding the signal power p_1. If an estimate of the clutter power is available from the clutter map, then we can subtract it from the total power to get p_1. If the signal *a* is weak and there is strong ground clutter as well as signal-*b* overlay, then to recover the signal-*a* velocity we need to apply the clutter filtering (Sachidananda and Zrnic 2000) as well as the overlay resolution algorithm. The effectiveness of this combination of algorithms has not been tested, mainly because it requires a large number of simulations. Moreover, the likelihood that ground clutter and overlaid echoes are simultaneously present is small; therefore, separation of the three has lower operational utility.

6. Conclusions

An algorithm is developed for resolving the one-overlay situation and estimating the spectral moments of both the overlaid signals in weather radar using the staggered PRT transmission scheme. The algorithm is tested on simulated time series and for the stagger ratio 2/3. The results of the simulation for typical S-band weather radar parameters and Gaussian-shaped spectra indicate that the recovery of the spectral parameters of the weaker of the two signals is effective if the overlaid signal has a narrow spectrum width. For the simulation parameters used, the weaker signal velocity can be estimated accurately at an overlay power ratio as high as 40 dB if the spectrum width of the overlaid signal is less than 3 m s^{-1}. However, the overlay resolution capability reduces sharply as the width increases. The staggered PRT pulse pair algorithm (without one-overlay resolution) works well up to an overlay power ratio of 3 to 6 dB. Thus, the overlay resolution algorithm needs to be used for large overlaid power cases only.

For a staggered ration of 2/3 the longer PRT, T_2 equals three basic time units T_u, and $T_1 = 2T_u$. Hence, with the one-overlay resolution the effective increase in unambiguous range is from T_1 to T_2 or 50%. An important observation is that the overlay resolution algorithm effectively filters the ground clutter while estimating the spectral parameters of the signal from ranges corresponding to time delays between T_1 and T_2 because the clutter signal has a narrow spectrum width. The clutter filtering is generally required only for the ranges smaller than $cT_u/2$.

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