

**SIGNAL DESIGN AND PROCESSING TECHNIQUES
FOR WSR-88D AMBIGUITY RESOLUTION**

PART - I

National Severe Storms Laboratory Report
prepared by: **M. Sachidananda**
with contributions by: **D.S. Zrnic and R.J. Doviak**

July 1997

NOAA, National Severe Storms Laboratory
1313 Halley Circle, Norman, Oklahoma 73069

SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

Part-I

Table of Contents

List of symbols	iii
1. Introduction	1
1.1. Range and velocity ambiguity	1
2. Simulation study	6
2.1. Weather radar signal simulation	6
2.2. The autocovariance algorithm	7
2.3. Procedure for evaluation of the algorithms	11
2.4. Programs	12
3. Peak Sorting Method	14
3.1. Introduction	14
3.2. Conceptual development	14
3.3. The peak sorting algorithm	20
3.4. Simulation study and results	21
3.5. Conclusions	24
4. Random phase coding	25
4.1. Introduction	25
4.2. Random phase coding and spectral parameter estimation	26
4.3. Choice of code	36
4.4. Some sample spectra and illustration of processing	39
4.5. The random phase algorithm	45
4.6. Simulation and results	47
4.7. Possible extension to 3rd and 4th trips	58
4.8. Conclusions	58
5. Systematic phase coding	59
5.1. Introduction	59
5.2. Systematic phase coding and spectrum modification	59

5.3. $\pi/4$ phase code	60
5.3.1. $\pi/4$ phase coding and spectral parameter estimation	60
5.3.2. The $\pi/4$ decoding algorithm	64
5.3.3. Simulation and results	65
5.4. $\pi/2$ phase coding	69
5.4.1. $\pi/2$ phase code and estimation of spectral moments	69
5.4.2. The algorithm development	71
5.4.3. The $\pi/2$ decoding algorithm	74
5.4.4. Simulation and results	80
5.5. Optimizing the systematic code	86
5.5.1. Conceptual development	86
5.5.2. The decoding algorithm for optimum systematic code	91
5.5.3. Simulation results and discussion	93
6. Summary and conclusions.	96
7. References	100

LIST OF SYMBOLS:

a_k, b_k, s_k, q_k	-	k^{th} spectral coefficient
c	-	speed of light
$C_{ab}(1)$	-	cross correlation of 1st and 2nd trip signals
C_k	-	complex modulation code [$C_k = \exp(j\phi_k)$]
$e1, e2$	-	complex time series of 1st and 2nd trips
$E1$	-	complex time series with 1st trip coherent and 2nd trip coded
$E2$	-	complex time series with 2nd trip coherent and 1st trip coded
E_i	-	complex time series samples
$\text{err}()$	-	error in the parameter in brackets
f_d	-	Doppler frequency
f_a	-	Nyquist frequency
G_k	-	spectral coefficients (fitted to the Gaussian shape)
i, k, n, m	-	used as indices
j	-	$(-1)^{1/2}$
M	-	number of samples
n_w	-	notch filter width normalized by $2v_a$
nw	-	notch filter width in terms of number of spectral coefficients
N_k	-	noise power in the k^{th} coefficient
$p1, p2$	-	mean power of 1st and 2nd trips
$pm1, pm2$ etc.	-	mean powers estimated from long PRT data
$pw1, pw2$	-	recovered 1st and 2nd trip powers (peak sorting algorithm)
P_k	-	power of the k^{th} spectral coefficient
r, r_c	-	range
r_a	-	unambiguous range
$R_a(1), R_b(1)$	-	autocorrelation of signals a and b
$r(k)$	-	random number array of length k
$R(n)$	-	autocorrelation for n PRT lag
R_p	-	residual power ratio (the ratio of power before notch filtering to the power after notch filtering, for the stronger signal)
R_o	-	overlapped power to total power ratio
S_k	-	power of the k^{th} spectral coefficient of the signal
$S1, S2$	-	spectrum of $E1$ and $E2$ [$S1 = \text{DFT}(E1)$]
T	-	pulse repetition time
v_r	-	radial velocity
v_a	-	unambiguous velocity (short PRT)
v_{al}	-	unambiguous velocity (long PRT)
v_m	-	mean velocity
$v1, v2$	-	mean velocity of 1st and 2nd trips
$vm1, vm2$, etc.	-	mean velocities from long PRT data

$vp1(i), vp2(i)$	-	estimated velocity aliases of the 1st and 2nd trips from long PRT data, $i=1,2,3$ & 4 (aliasing interval number)
$w1, w2$	-	spectrum width of 1st and 2nd trips
z	-	$\exp(j2\pi/M)$
$\hat{}$	-	estimate
\mathcal{P}	-	probability
\mathcal{E}	-	expected value
τ	-	range time
Ψ_k	-	switching phase sequence
Φ_k	-	modulation phase sequence

ABBREVIATIONS:

SNR	-	Signal-to-Noise Ratio
PRT	-	Pulse Repetition Time
GCF	-	Ground Clutter Filter
DFT, IDFT	-	Discrete Fourier Transform, Inverse DFT
FFT	-	Fast Fourier Transform
S&S	-	Smoothing and Subtraction
$\pi/4$ code	-	{ 0 , $\pi/4$, 0, $\pi/4$, ... }
$\pi/2$ code	-	{ 0 , 0, $\pi/2$, $\pi/2$, ... }

SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART - 1

1. INTRODUCTION

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL), the National Center for Atmospheric Research (NCAR), and the Forecast Systems Laboratory (FSL) to address the mitigation of range and velocity ambiguities in the WSR-88D system. This is the first part of the report on the ambiguity resolution in WSR-88D. It documents the work done at the NSSL in the first year of the project. Selected techniques that rely on spectral processing to sort out overlaid echoes are investigated, and the best candidate is selected for further scrutiny.

1.1. Range and velocity ambiguity

The range to scatterers is found by measuring the time delay between a transmitted pulse and its echo. If a transmitted pulse is part of an equi-spaced pulse train, the measured range is ambiguous because the echo signal could be due to any one of the pulses transmitted earlier (Doviak and Zrníc 1993). Therefore, the measured range, r , for a delay time, τ , is given by

$$\begin{aligned} r &= (n-1) cT/2 + c\tau/2 ; \quad 0 \leq \tau \leq T \\ &= (n-1) r_a + r_\tau , \end{aligned} \tag{1.1}$$

where T is the pulse repetition time, c is the speed of light, r_a is the unambiguous range, and n is an integer or the trip number. Range ambiguity resolution is the determination of the trip number, n . Equation (1.1) implies that the echo measured at delay time τ could be due to any one of the pulses transmitted earlier, or in other words, the echo is due to scatterers located at any of the range cells corresponding to the time delay, $[(n-1)T + \tau]$. One simple method of determining the correct range is to choose T large enough to make $r_a = cT/2$ encompass ranges beyond which radar beam is about 16 km above ground so that no storms are intercepted, and $n=1$ can be assumed safely. However, this is not an acceptable solution because it severely curtails the velocity measurement capability of the radar.

The radial velocity of scatterers is obtained from the measurement of Doppler frequency,

f_d . The radial velocity of the scatterer is related to the Doppler frequency by

$$v_r = - \lambda f_d / 2 , \quad (1.2)$$

where λ is the free-space wavelength. By convention, scatterers moving away from the radar have a positive velocity which produce negative Doppler shift. Because the echoes are discrete samples taken at intervals, T , the maximum Doppler frequency that can be unambiguously extracted from the sample sequence is given by

$$f_a = 1/(2T), \quad (1.3)$$

which is known as the Nyquist frequency. A fully coherent radar can recover Doppler frequencies within the interval $\pm f_a$. Any frequency outside this interval is seen by the processor as a measured Doppler frequency, f_d , within the aliasing interval such that

$$f_d - f_{da} = \pm 2 m f_a. \quad (1.4)$$

Here, f_{da} is the actual Doppler frequency. The integer, m , is the aliasing interval number. Therefore, the actual Doppler frequency is known only within an unknown integer, $\pm m$. Corresponding to the unambiguous frequency interval, $\pm f_a$, the unambiguous velocity interval is $\pm v_a$, where $v_a = \lambda/4T$. Since both v_a and r_a are functions of pulse repetition time, T , we can combine them to get

$$r_a v_a = c \lambda / 8. \quad (1.5)$$

Thus, if r_a is increased by increasing T , v_a decreases correspondingly. This is a fundamental limitation of a pulsed Doppler radar transmitting uniformly spaced pulses. However, the problem is overcome if means are found to determine the trip number, n , and the aliasing interval number, $\pm m$.

Discussions so far assume that the scatterers are located at only one of the several ambiguous locations corresponding to the delay, $[(n-1)T+\tau]$, and are moving with a certain radial velocity. In a pulsed Doppler weather radar, the situation is somewhat more complicated because the scatterers are precipitation particles that can be distributed quasi-continuously over a large area ($> 10^2 \text{ km}^2$), and the dynamic range of the echo strength can be as large as 80 dB. Therefore, the echo sample can consist of echoes from more than one ambiguous range cell. If this is the case, the signals are said to be overlaid. Typical radial velocities encountered in storms can span a $\pm 50 \text{ m s}^{-1}$ interval (Doviak and Zrnicek 1993, p. 165, Fig. 7.4). The unambiguous range requirement for a 10 cm wavelength weather radar, such as WSR-88D, is about 460 km. Because of the curvature of the earth, the antenna beam would top most of the storms at about this range. If T is chosen to obtain $r_a=460 \text{ km}$, the unambiguous velocity interval is too small to effectively de-alias velocities using data processing techniques. On the other hand, if T is chosen small enough to give at least $\pm 22 \text{ m s}^{-1}$ unambiguous velocity, the echo signals from different range cells, corresponding to different trip numbers, n , may be overlaid.

In this case, one needs to separate the signals and then estimate their spectral parameters. Therefore, in the case of weather radar, ambiguity resolution must also include signal separation, in addition to the determination of trip number n and aliasing interval number, m .

In general, it may not be necessary to determine both m and n because if there is a technique to determine the trip number, n , one can choose a T short enough to make v_a encompass all the expected values of velocities so that m can be safely taken to be zero. Similarly, if there is a technique to find the aliasing interval, then T could be increased to yield a larger r_a . But echoes from subsequent pulses become less correlated as T increases, and therefore, spectral moment estimates deteriorate (Doviak and Zrnich 1993). At a 10 cm wavelength and a spectrum width of weather signals of 4 m s^{-1} (the median in severe storms), the decorrelation time (lag at which the correlation is $e^{-1/2}$) is about 2 ms, and the corresponding $r_a = 300 \text{ km}$. Therefore, overlaid echoes are inevitable, and some kind of pulse to pulse coding (in time, frequency, phase, amplitude, polarization, etc.) or some a-priori information about the echoes, must be used to sort them.

Several methods which seek to ameliorate the problem of range and velocity ambiguity have been reported in the literature. Notable among them are (a) staggered PRT scheme (Zrnich and Mahapatra 1985) which can be considered as time coding, (b) random phase coding (Zrnich and Mahapatra 1985), (c) systematic phase coding (Sachidananda and Zrnich 1986), and (d) polarization coding (Doviak and Sirmans 1973). A radar that alternately transmits horizontally and vertically polarized waves can increase the unambiguous range by a factor of 2 (Zahrai and Zrnich 1993).

All these methods fall under the category of signal design and processing. There are other techniques developed to obtain aliasing interval information based on the continuity of velocity fields and/or knowledge of environmental winds. These techniques are implemented after fields of velocity are collected and can be classified as data field processing techniques.

The WSR-88D transmits pulses at two PRTs on sequential azimuthal scans or adjacent radials. The long PRT mode is for estimating the reflectivities over a large range, and the short PRT is for velocities over a smaller unambiguous range interval. In this scheme, unambiguous reflectivity information obtained in the long PRT scan is used to assign appropriate trip numbers, n , to signals in the short PRT scan. But in the case of overlaid echoes, only the stronger one is recovered in the WSR-88D signal processor. Further spectral processing could often recover the weaker echo as well. For example, the signal in the short PRT scan can be analyzed to locate peaks in the spectral domain and assign appropriate ranges to the corresponding velocities using the reflectivity information available from the long PRT scan. Such techniques fall in the category of spectral peak sorting, or simply peak sorting.

This report deals with signal design and processing techniques and examines methods that utilize a uniform PRT sequence with a sufficient number of samples for meaningful Fourier analysis (typically 64). Specifically, the following three are studied: (a) peak sorting, (b) random phase coding, and (c) systematic discrete phase coding. An in-depth simulation study has been carried out to evaluate their potential for possible implementation on the WSR-88D radar.

Except for peak sorting, the other two methods fall under the category of pulse to pulse coding or modulation. The main idea behind coding is to imprint some signature on the return signals corresponding to different transmitted pulses. These signatures are used to decode or separate the signals belonging to different trips. The methods (b) and (c) are different to the

extent that the code sequence is chosen differently, and an appropriate decoding scheme is evolved based on the properties of the modulated spectrum.

The report is organized in the following manner. In section 2, weather signal simulation procedure is discussed, along with the methodology for evaluating the algorithms developed for range and velocity ambiguity resolution in the presence of overlaid echoes. Then, in the subsequent three sections, several algorithms are discussed including the simulation results on the performance of the algorithms. In simulating the weather signal time series and in comparison of the methods of ambiguity resolution, the following assumptions are made:

- a) **the ground clutter is absent**
- b) **the noise is absent**
- c) **the window effect is absent**
- d) **the spectra have a Gaussian shape**
- e) **only the 1st and 2nd trip signals are present in the time series.**

The decoding algorithms presented in this report are also developed with these assumptions. In practice, several of these effects, such as the window effect, the ground clutter, the noise, etc., are present and have to be accounted for in the algorithms. However, these are common to all, and would affect all the methods in some way or other in limiting their performance; hence they were neglected in evaluating the relative performance of the methods in order to identify the best one. These assumptions help bring out the basic capability of the methods which otherwise would be masked by some of these effects. The effect of the signal-to-noise ratio, the window effect, the ground clutter, and many more practical aspects of implementation of the algorithm on the WSR-88D are analyzed in Part 2 of this report where the selected method is scrutinized in much greater detail.

The WSR-88D in the present configuration can be operated in two different volume coverage patterns (vcp-11 and vcp-21), and in both, several preset PRT values are used which can be grouped as long PRT and short PRT. The long PRT is about 3 ms, and the short PRT is selected from a pre-defined set of values between 0.7 and 1 ms. The long PRT scan is called the surveillance scan (CS) and the short PRT scan is called the Doppler scan (CD). The volume coverage includes an azimuth scan of 360° at discrete elevation angles from 0.5° to 19.5° degrees. At the lowest two elevations (0.5° and 1.45°), two scans each (CS and CD) provide reflectivity to about 460 km and velocity to about 115 km, if there are no overlaid echoes. The number of samples available for spectral parameter estimation is about 16 to 17 samples in the long PRT scan and 44 to 66 samples in the short PRT scan, in the vcp-11 mode. In the vcp-21 mode, the number of samples is 28 for the CS scan and 75 to 111 for the CD scan.

In the scans at elevations 2° to 6.5° , the long and short PRT are used in alternate radials in each scan, called batch mode (B). The number of samples available for spectral parameter estimation is 6 to 10 with the long PRT, and 35 to 50 with the short PRT, in vcp-11. The corresponding numbers are 8 to 12 and 59 to 88 for vcp-21. Above 6.5° elevation, only the short PRT Doppler scan is used.

In the lower elevation scans ($< 6.5^\circ$), the velocity recovery is hampered by overlaid echoes. The aim of this study is to evolve a solution to this problem. From the assumption (e) given above, it is obvious that the ambiguity resolution methods considered can, at best, extend

the range to only two trips. And if the unambiguous velocity is to be near 35 m s^{-1} , this extended range would be about 230 km. In the two lowest elevation scans, the unambiguous range requirement is about 460 km for the reflectivity; therefore, the long PRT scan must be retained. This leads us to the possibility of using the information from the long PRT scan data in ambiguity resolution algorithms applied to short PRT data. However, in the intermediate elevation scans (2° to 6.5°), the antenna beam tops a typical storm within the first two trips (about 230 km). Hence, there may be a possibility of replacing the batch scan with the Doppler scan, if the algorithm is able to extract all three spectral parameters of both the 1st and the 2nd trip echoes, with the required accuracy. Therefore, the methods of ambiguity resolution presented in this report are developed as stand-alone methods (not using long PRT data), whenever possible. But some methods are based on the availability of the long PRT data and cannot operate in a stand-alone mode. This is indicated in each algorithm.

2. SIMULATION STUDY.

2.1. Weather radar signal simulation.

In order to test the effectiveness of techniques for mitigating the range and Doppler ambiguities, it is desirable to make tests on overlaid radar signals. Comprehensive tests are necessary but are practical only on simulated radar signals because only with simulation can the actual individual signal parameters (i.e., the mean power, mean velocity, and spectrum width) be accurately known. The recovered parameters can be compared with the specified input parameters to determine the error in the estimates. However, the inferences drawn by this study can be meaningful only if the simulated weather spectra truly represent the radar signals from storms. Zrnic (1975) gives a procedure to simulate the weather signal on a digital computer using a random number generator available in most computers. The complex sample, E_i , in a time series can be expressed as a discrete Fourier series;

$$E_i = 1/M \sum_{k=0}^{M-1} P_k^{1/2} \exp(j \theta_k) \exp(j2\pi ki/M). \quad (2.1)$$

Here, P_k is the exponentially distributed instantaneous power of the signal plus noise in the k^{th} spectral coefficient. The signal part is frequency dependent, and the noise part is white; θ_k is a uniformly distributed phase; and P_k and θ_k are statistically independent. With S_k as the signal power and N_k as the noise power, in the k^{th} coefficient P_k , the probability density of P_k can be written as

$$\mathcal{P} \{P_k\} = 1/(S_k+N_k) \exp[-P_k/(S_k+N_k)]. \quad (2.2)$$

This is the basic equation used in the simulation of weather spectra. The steps include the generation of a Gaussian shaped S_k and adding a noise to get the desired signal-to-noise ratio (SNR). These coefficients are multiplied by the logarithm of a uniformly distributed random number (0 to 1) to get P_k . The phases, θ_k , are generated from the same uniform number generator but are independent.

This procedure was followed to generate weather signal spectra, and the inverse discrete Fourier transform (IDFT) was used to obtain the time series. To simulate the window effect, a very long time series is generated, and a short part, of length M , is taken and multiplied with appropriate weights. The window effect is not very critical in the evaluation of algorithms because it increases the spectrum width by a small amount. Simulating the window effect takes more computer time because long time series need to be generated. Therefore, for initial evaluations, the window effect was not included. The velocity aliasing is simulated by generating a time series without aliasing and then selecting alternate samples to simulate one-time aliases. Multiple aliasing is simulated by dropping n samples after each selected sample.

The program developed for time series simulation, "TSR1.M", has mean power, velocity, spectrum width, and number of samples, M , as the inputs. The output is a time series of complex samples (i.e., in-phase samples, I , and quadrature samples, Q) of length M . Three other

