

**SIGNAL DESIGN AND PROCESSING TECHNIQUES
FOR WSR-88D AMBIGUITY RESOLUTION**

PART 2

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LIST OF SYMBOLS:

c	-	speed of light
C_k	-	modulation code [$C_k = \exp(j\phi_k)$]
e_1, e_2	-	complex time series of 1st and 2nd trips
E_1	-	complex time series with 1st trip coherent and 2nd trip coded
E_2	-	complex time series with 2nd trip coherent and 1st trip coded
E_i	-	complex time series samples
$err()$	-	error in the parameter in brackets
i, k, n, m	-	used as indices
j	-	$(-1)^{1/2}$
M	-	number of samples
n_w	-	normalized notch filter width (normalized to $2v_a$)
p_1, p_2	-	mean powers of the 1st and 2nd trip signals
p_n	-	noise power
p_c	-	clutter power
P_k	-	power spectral coefficients
p_{coh}	-	coherent part of the power
p_{inc}	-	incoherent part of the power
r_a	-	unambiguous range
$r(k)$	-	random number array of length k
$R(n)$	-	autocorrelation for n pulse lag
R_c	-	clutter rejection ratio
R_p	-	residual power ratio (ratio of the power, p_1 , to the residual $(p_1)_r$ after notch filtering)
R_{pe}	-	effective residual power ratio with random phase error in the phase shifter
R_{pt}	-	residual power ratio (ratio of the power (p_1+p_c) , to the residual power of $(p_1+p_c)_r$ after notch filtering)
R_o	-	overlapped power to total power ratio
s_k	-	k^{th} complex spectral coefficient
$sd()$	-	standard deviation of the parameter in brackets
S_1, S_2	-	spectrum of E_1 and E_2 [$S_1 = \text{DFT}(E_1)$]
T	-	pulse repetition time
v_a	-	unambiguous velocity
v_m	-	mean velocity
v_1, v_2	-	mean velocities of the 1st and 2nd trip signals
w_1, w_2	-	spectrum widths of the 1st and 2nd trip signals
w_c	-	clutter spectrum width
w_{cf}	-	clutter filter width
z	-	$\exp(j2\pi/M)$
$\hat{}$	-	estimate

\mathcal{P}	-	probability
\mathcal{E}	-	expected value
τ	-	range time
ψ_k	-	switching phase code sequence
ϕ_k	-	modulation phase code sequence
μ	-	ratio of residual powers before and after subtraction

ABBREVIATIONS:

CD	-	Contiguous Doppler
CNR	-	Clutter-to-Noise Ratio
CS	-	Contiguous Surveillance
DFT	-	Discrete Fourier Transform
FFT	-	Fast Fourier Transform
GCF	-	Ground Clutter Filter
IDFT	-	Inverse Discrete Fourier Transform
PNF	-	Process Notch Filter (notch filter in the SZ decoding algorithm)
PRT	-	Pulse Repetition Time
SCR ₁	-	Signal-to-Clutter ratio (1st trip signal)
SCR ₂	-	Signal-to-Clutter ratio (2nd trip signal)
SNR	-	Signal-to-Noise Ratio
SNR ₁	-	SNR of the 1st trip signal ($=p_1/p_n$)
SNR ₂	-	SNR of the 2nd trip signal ($=p_2/p_n$)
$\pi/4$ code	-	{ 0, $\pi/4$, 0, $\pi/4$, ... }
$\pi/2$ code	-	{ 0, 0, $\pi/2$, $\pi/2$, ... }

SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART 2

1. INTRODUCTION

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL), the National Center for Atmospheric Research (NCAR), and the Forecast Systems Laboratory (FSL) to address the mitigation of the range and velocity ambiguities in the WSR-88D system. This is Part 2 of the report on the ambiguity resolution. It documents the work done at the NSSL which was completed in the second year of the project.

In the first part of this report, several ambiguity resolution algorithms were studied theoretically, and extensive simulations were carried out to evaluate their performance. A comparison of the capabilities of each of these algorithms led to the selection of a systematic phase coding technique (the SZ code) as the best among the methods considered. In this second part of the report, we examine the properties of the SZ code in more detail, especially related to the processing steps used in the decoding algorithm. Several important practical aspects which were not included in the earlier simulation, and some aspects of implementation of the algorithm on the WSR-88D, are also included.

Some of the effects present in a practical radar signal are the window effect, ground clutter, and receiver noise. These are addressed here, and their contribution to the degradation in the performance of the systematic phase coding scheme is evaluated. A new and important consideration that needs scrutiny is the accuracy of the phase shifts that has to be maintained for the effective operation of the phase coding scheme. An analysis of the effect of random errors in the phase shifter on the performance of the algorithm, especially the velocity recovery, is carried out, and the results are presented. These practical aspects of the radar signal affect the performance of the different algorithms with respect to the recovery of the weaker signal velocity. Therefore, although the SZ(8/64) code was selected to be the best code in the first part of this report, we also study SZ codes with alternative values of (n/M) , and with appropriately modified decoding algorithms, in an effort to evolve a phase coding scheme which performs best with all the practical constraints in place.

Another important practical consideration is the compatibility of the phase coding scheme with the present scan strategy of the WSR-88D. There are several parameters, such as the scan rate, the PRT, the number of samples, etc., that are pre-set in the WSR-88D, depending on the operating mode. The points to be addressed are: (a) how many of these parameters need to be changed in order to implement the phase coding scheme, and (b) how to integrate the phase coding scheme in the WSR-88D so that the meteorological performance of the radar is not

compromised. Ideally, one would like to make the least amount of changes. However, if a change leads to a significant advantage without compromising existing capabilities, it should be made. All these, and many finer points of practical implementation of the algorithm, are discussed in this report.

With the implementation of the coding scheme to recover velocities of the first two trip echoes, we would have a range coverage of about 230 km for $v_a = 32 \text{ m s}^{-1}$ with a transmitting frequency of 3 GHz. The requirement of the reflectivity data over a range of 460 km at low elevation angles makes it imperative that we retain the present long PRT scan (Contiguous Surveillance mode of the WSR-88D). Some of the information obtained from this long PRT scan can be used in the short PRT Doppler scan (Contiguous Doppler mode) data processing to improve, speed up, and channel the computations along different paths in the algorithm.

At the intermediate elevation angles (2.5° to 6.5°), the batch mode of data acquisition is used, in which alternate radials have long and short PRT transmissions. Here also, the information obtained from the long PRT data can be used in the processing of the short PRT data. However, a change in the scanning mode of the WSR-88D is possible for these elevation angles because the lower edge of the beam at 2.5° elevation is above 11 km at ranges beyond 230 km. Thus, practically all of the storms detected at elevation angles 2.5° and above will be within 230 km, the range to which Doppler velocity processing is required. But we show that the SZ phase coding scheme can recover all the three parameters (i.e., if $p_1/p_2 < 40\text{dB}$) over twice the unambiguous range interval without the need for long PRT data. Thus, above 2.5° elevation, we can increase the data acquisition rate which can be used to decrease the scan time. Because of this possibility, we have developed the decoding algorithm in two different forms (SZ-1 and SZ-2). The first one works in a stand alone mode to recover all three spectral parameters of both trips, and the second algorithm recovers only the velocities of both trips and uses the long PRT scan data for the recovery of the reflectivity and the spectrum width. (The long PRT data in the present WSR-88D is used to estimate reflectivity only.) The second algorithm can be used if the long PRT data is available, and the first algorithm can be used if the batch mode is replaced by a phase coded Doppler scan.

Throughout this report, we use the following assumptions and notations for convenience: it is assumed that only the 1st and 2nd trip echoes are present in the radar signal, and the first trip is always stronger. The signal parameters, viz., the mean power, the mean velocity, and the spectrum width, are represented by p_i , v_i , and w_i , respectively, with the subscript $i=1,2$ representing 1st or the 2nd trip. Generally, the stronger signal parameters are easily recovered with the phase coding scheme; the limitation is in the recovery of the velocity of the weaker signal. Therefore, most of the discussions will be about the recovery of the velocity, v_2 , of the weaker signal. When the spectral parameters are estimates, obtained from the time series (or spectrum), the symbol $\hat{}$ is used to represent estimates. However, for convenience, this symbol has been omitted in many places. But it is clear from the context whether the parameter is an estimate or not.

In the lowest two elevation scans there is a possibility of multiple trip echoes. However, the SZ algorithm is developed for recovery of the first two trip signal parameters in the absence of the 3rd and 4th trip echoes. The multiple trip overlay case is discussed in some detail in section 5.6.

In Part 1 of this report, the error in the spectral parameters, estimated using the SZ decoding algorithm, were computed with respect to the autocovariance estimates of the same parameters obtained from the individual time series before they are combined to form the overlaid signal time series. This was done specifically to present the performance of the algorithms relative to the autocovariance processor, so that the comparison among the algorithms is made easier. This can give a false impression that the estimation error is zero in some cases (e.g., $err(\hat{v}_j)$ is zero for large p_1/p_2). Results presented in this report use errors computed with respect to the nominal parameters of the simulation, so that they represent realistic errors.

In Section 2, a comprehensive discussion on the SZ phase code, vis-a-vis the processing steps in the decoding algorithm, is presented. Section 3 is a detailed study of the various effects which are normally present in weather signals. Specifically, we address the effects of various window functions, white noise, and ground clutter filtering. Several practical aspects, which are specific to the phase coding scheme and the WSR-88D, such as the errors in the phase shifter and its effect on the performance of the phase coding scheme, the sample length selection, the code synchronization, etc., are discussed in Section 4. The two versions of the SZ decoding algorithms are given in Section 5 along with some results on the overall performance of the SZ phase coding schemes. Specifically, the performance of three SZ codes, viz., the SZ(8/64), SZ(12/64), and the SZ(16/64), is discussed and compared to arrive at an optimum code with all the practical effects included. A proposed schematic of the algorithm for implementation on the radar is also discussed in this section.

Whereas the simulated time series is a very good tool in the design stage of the phase coding scheme, it cannot represent all the variations in the actual radar derived time series. Due to the diversity in the weather phenomenon, there are situations which produce a non-Gaussian shaped spectra. Therefore, the actual performance of the phase coding scheme has to be obtained by testing it on real weather signals. This will be investigated in Part 3 of the report, which will be devoted to the study of the WSR-88D data. Here, simulated data fields are generated with overlaid echoes to demonstrate the performance of the algorithm. These results are discussed in Section 6 of this report. The conclusions drawn from this study are in the last section.

2. SZ PHASE CODING SCHEME

In a phase coded radar, the transmitter pulses are phase shifted by a pre-determined phase sequence, ψ_k , and the received echo samples are phase corrected (multiplied by $\exp\{-j\psi_k\}$) so that the 1st trip signal is coherent. However, the 2nd trip echo would not be coherent but will be modulated by a phase sequence $\phi_k = (\psi_{k-1} - \psi_k)$. If the second trip is coherent, then the 1st trip echo is modulated by a phase sequence $-\phi_k$. Here, ψ_k is the **SZ switching code** (phase shifter switching sequence), and ϕ_k is the **SZ modulation code**. In autocovariance processing, the mean velocity is estimated from the phase of the autocorrelation for lag 1, $R(1)$. The modulation code, ϕ_k ($-\phi_k$), modifies the spectrum of the 2nd (1st) trip echo so that its $R(1)$ is made zero; thus, the bias error in the velocity estimate of the coherent 1st (2nd) trip signal, due to the overlaid 2nd (1st) trip signal, is removed. For a given spectrum width, the variance of the velocity estimate increases directly with the increase in the overlaid power, and the estimated velocity is usable when the modulated overlaid power is less than the coherent power (i.e., equivalent to 0 dB SNR, considering the modulated power as noise). Therefore, further processing is needed to remove as much of the overlaid power as possible from the spectrum so that the ratio of the coherent signal power to the residual overlaid modulated signal power is greater than unity. This is accomplished by the notch filtering and cohering steps in the decoding algorithm. There is a certain amount of self-noise (Zrnic and Mahapatra, 1985) generated in the process of notch filtering and cohering which results in a decreased SNR. The self-noise power is a function of the code, the notch filter width, and the spectrum width. The SZ code is designed to allow the removal of the maximum amount of overlaid power, and at the same time minimize the self-noise, to improve the recovery of the velocity of the weaker signal.

2.1. Properties of the SZ phase code.

The modulation phase sequence is given by $\phi_k = n\pi k^2/M$, where M and n are integers (modulation code is $\exp\{j\phi_k\}$). If M is not divisible by n , this code has a property of zero autocorrelation for all lags except zero or multiples of M . This code was reported in a correspondence by Chu (1972). To modulate the 2nd trip signal with this code, if the 1st trip is made coherent, the transmitted phases have to be

$$\psi_k = - \sum_{p=0}^k \{n\pi p^2/M + \text{const.}\} ; k=0,1,2,\dots \quad (2.1).$$

The constant is arbitrary and is set to zero. Another important property of the SZ modulation code is that its autocorrelation (as explained earlier) and power spectrum are independent of a shift in the code (i.e., in Eq. 2.1, k values can be from m to $m+M-1$, with arbitrary m). We refer to this code as **SZ(n/M) code**. Note that the symbol M is also used for representing the number of samples in the time series, and we consider values of n less than $M/2$ only. The reason for considering $1 \leq n \leq M/2$ is that the modulation phase code repeats after $n=M/2$, except for a shift and/or conjugation. For a given M and $n=x$, $n=iM-x$, and $n=iM+x$, the modulation codes are

essentially the same for any integer i except for a conjugation and/or shift by integer multiples of $\pi/2$. The reason for choosing the parameter M to be the same as the number of samples is that M/n is the basic periodicity of the modulation code (if M/n is an integer), and for effective operation of the phase coding scheme, it is required to limit the periodicity to M or less. The indicated choice automatically limits the periodicity of the modulation code to sub-multiples of the number of samples. The number of samples available in the WSR-88D is between 44 and 66; hence, $M=64$ is selected as a convenient number for most of the computations in this report.

In general, for $M=64$ and any n , the periodicity of the modulation code can be obtained by expressing $M/n = P/q$, with all common factors between M and n removed such that q is an odd integer (i.e., equivalent to Chu's code $\phi_k = q\pi k^2/P$). P is the periodicity of the modulation code, and $4P$ is the periodicity of the switching code. Thus, it can be seen that by choosing M to be the same as the number of samples, we are restricting the periodicity of the modulation code to M or less. The autocorrelation is unity for lags in multiples of P . The spectrum of the code has only P non-zero coefficients spaced M/P coefficients apart. If a weather signal time series is multiplied by the modulation code, the spectrum of the resulting time series is a convolution of the code spectrum and the signal spectrum. If the signal spectrum is unimodal, it is easy to visualize that for $n=1$, the modulated spectrum is noise-like, and for $n=32$, it is bimodal (see Fig. 5.1, for $\pi/4$ code, Part 1 of the report). For $n=1$, the noise-like spectrum yields $R(1)=0$ in the mean, but there is an upper limit for the suppression of $R(1)$ by modulation, for any given realization of the signal time series with a practical number of samples. However, for $n=32$, the bimodal spectrum yields much better suppression of $R(1)$ because of the matching property, which is obtained for each realization. The matching property, as discussed in Part 1 of this report, is the equality of the k^{th} and $(k+M/2)^{\text{th}}$ spectral coefficient magnitudes ($|s_k| = |s_{k+M/2}|$; $k=1,2,\dots,M/2$). Only the difference power ($|s_k|^2 - |s_{k+M/2}|^2$) contributes to $R(1)$. As n is increased from 1 to 32, the whitening property gradually changes to matching property.

From the results presented in Part 1 of this report, it is observed that the region of recovery of v_2 in the $\{p_1/p_2; w_1\}$ space is approximately demarcated by the residual power ratio, R_p , (for definition of R_p see list of symbols, page ii) for the notch filter width used in the decoding algorithm. The $sd(\hat{v}_2)$ in the region of recovery is dictated by the overall SNR that is achieved for the weaker signal after the notch filtering and cohering steps. With a larger value for the code parameter n , better overall SNR can be achieved, but with a smaller notch width, which reduces the region of recovery of v_2 in $\{p_1/p_2; w_1\}$ space. Although earlier simulation study (Part 1 of the report) indicated the SZ(8/64) code as the best, it is not necessarily the optimum when the window and the noise effects are included. In Fig. 5.13 of Part 1 of this report, the notch width was fixed at $n_w=0.75$, which is not the optimum for all n values. In fact, for a given n , there is a maximum value of n_w beyond which the cohering process breaks down. This limiting value of normalized notch width can be written as

$$(n_w)_{\max} = |1 - 2n/M|; \quad 1 \leq n \leq M. \quad (2.2)$$

This limiting value of n_w is derived from the fact that the modulation spreads the power in each of the spectral lines of the signal into M/n spectral lines separated by n coefficients, and at least two of these spectral coefficients are required for cohering the signal without the loss of the mean velocity information. Assuming that the notch width is within the maximum limit, increasing n_w

