SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART - 3

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1. Introduction.

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL), the National Center for Atmospheric Research (NCAR), and the Forecast Systems Laboratory (FSL) to address the mitigation of range and velocity ambiguities in the WSR-88D. This is the third report in the series that deals with velocity and range ambiguity resolution in the WSR-88D. The first two reports mainly dealt with uniform PRT transmission and phase coding techniques to resolve the range ambiguity. Although the phase coding techniques do not directly address the velocity ambiguity problem, its capability of separating overlaid echoes allows the use of shorter PRTs which, in turn, diminishes the occurrence of ambiguous velocities. In this third part, we consider the staggered PRT technique and its variants. Because the pulse repetition time is varied in all these schemes, we classify these methods as time coding methods. A comparison of the SZ phase coding technique and the time coding techniques is also carried out to arrive at a best strategy to be adopted for ambiguity resolution in the WSR-88D.

There are several variants of the time coding schemes reported in the literature. Notable among them are (i) staggered PRT technique (Zrnic and Mahapatra, 1985), (ii) spaced pair with polarization coding (Doviak and Sirmans, 1973), (iii) interlaced sampling (Sirmans et. al. 1976; Doviak and Zrnic, 1993), (iv) multi-PRI scheme suggested by Lincoln Lab, MIT, (Chornoboy and Weber, 1994), (v) alternating codes suggested by Finnish scientists (Pirttila et. al., 1999). A detailed discussion on some of the methods is available in the book by Doviak and Zrnic (1984). Conceptually, some of these methods can provide a large unambiguous velocity and range. However, their practical utility has been limited by three major problems: (a) the difficulty in filtering the ground clutter, (b) resolving overlaid echoes, and (c) obtaining spectral moment estimates with a reasonably low variance. The clutter filtering introduces unacceptably large bias error in velocity estimates in certain Doppler bands, and the overlaid echoes increase the variance of the spectral estimates.

Most promising among the schemes listed in the previous paragraph are the staggered PRT schemes, and they are possible candidates at least for the higher elevation scans in the WSR-88D. Therefore, in addition to examining all the available schemes, extra effort was put in solving the outstanding problems of the staggered PRT schemes; viz., the clutter filtering and improving the variance of the estimates. We shall focus on the staggered PRT scheme in which 2 PRTs are alternately transmitted.

At least half a dozen different ways of processing the staggered PRT samples were examined using simulation procedures, before a successful solution was arrived at. All these methods are not included in this report, but only the successful method has been given. Here we briefly recount the story of investigations that led to the successful solution to the clutter filtering problem. The decreased variance of the spectral moment estimates was an additional gain that results from the new procedure.

To begin with, we started from an examination of the available results in Zrnic and Mahapatra (1985) and Banjanin and Zrnic (1991). Banjanin and Zrnic (1991) have used a pair of filters on uniform sample sequences derived from the staggered PRT sequence. Torres (1998) has investigated the use of an orthogonal polynomial based regression filter on the non-uniform staggered PRT samples directly. These efforts have indicated that the rejection bands are not easily removed. This is because the problem is in the non-uniform sampling itself and not in the clutter filtering method. The non-uniform sampling produces aliasing of some of the signal power into the clutter spectral regions, and hence, these portions are indistinguishable from the clutter.

The rejection bands in the clutter filter can be traced to the periodicity in the staggered PRT sampling scheme. Therefore, our first approach was to remove the periodicity in the sampling wave form with the hope that this will remove the rejection bands in the clutter filter or smear the rejection bands over the entire spectrum while greatly reducing the attenuation. There are two ways this could be done: (i) vary the PRTs randomly about some mean value, or (ii) switch between the two PRTs, T_1 and T_2, randomly. In the first case, computation of autocorrelation is complicated; hence, the second method was studied using simulation. It was observed that with randomly switched PRTs, the variance of the velocity estimate is higher ($sd(v)$ was 2 to 3 times larger), and the clutter filter rejection bands do spread out and have a much lower attenuation. The rejection level could be reduced to almost 1dB and spread throughout the spectrum, but at the expense of an increased error in the velocity estimate.

In order to improve the error performance, next we considered spaced pair transmission. The pulse spacing T_1 is made small so that the velocity can be computed from $R(T_1)$ directly rather than using the difference, as in the staggered PRT scheme. Although it is well known that signal overlay cannot be avoided in this case unless polarization switching is used, we encounter other problems such as isolation between the two channels, differential propagation phase shift, etc., which need to be considered in evolving an algorithm for spectral moment estimation. Therefore, the spaced pair scheme was considered without the polarization switching, and with the hope that some kind of phase coding can be worked out to separate the overlaid signals once an effective way of clutter filtering is designed. The advantage seen here is the lower $sd(v)$ available with a spaced pair.

A successful clutter filtering procedure was developed for the spaced pairs scheme, which involved polynomial interpolation to reconstruct an equivalent uniform PRT sequence from the spaced pair sequence of complex samples. Since the clutter signal is of low frequency, the reconstruction is very good; therefore, a spectral domain filter can reduce the bias error in the velocity estimate to a reasonably low value while still maintaining a low $sd(v)$. However, we could not make much headway in separating the overlaid signal with phase coding, without which the spaced pairs scheme is not useful in practice.

The polynomial interpolation was then tried on the staggered PRT sample sequence, but with much less success. The clutter filtering was effective, but the rejection bands in the frequency response of the clutter filter, which produced large bias errors in the velocity estimate, were still present. In all the simulation studies of the clutter filtering, it was observed that the bias error is systematically negative for positive velocities and positive for negative velocities. It was logical to look for a clutter filtering scheme which produced an exactly opposite bias so that it can be canceled. The result of this search produced the new algorithm.

A major turning point in the search for a clutter filtering method occurred when we started looking at the staggered PRT signal in the spectral domain. This gave us an insight into the spectral properties of the staggered PRT signal which subsequently lead to the development of an effective clutter filtering scheme and also resulted in an improvement in the spectral moment estimates. Thus, two problems were solved at the same time. The new algorithm developed for the staggered PRT scheme overcomes the problem of clutter filtering completely. The algorithm also estimates the spectral moments with much lower variance than that reported earlier. With this new algorithm, it becomes practical to use the staggered PRT scheme to achieve a larger unambiguous velocity and range with acceptable error levels in the Doppler spectral moment estimates.

Although the study was started with an intention of an in depth examination of other available time coding schemes (e.g., the block staggered, spaced pair with polarization switching, etc.) to evaluate their suitability to the WSR-88D ambiguity problem, the discovery of the new clutter filtering and spectral processing method, whose performance surpassed all our expectations, has prompted a change in the emphasis so that a major part of the report is devoted to the delineation of the new algorithm for the staggered PRT scheme and its performance.

2. The staggered PRT technique.

Here, we describe the staggered PRT scheme briefly before we embark on a discussion of the new method of processing. In the staggered PRT technique (Zrnic and Mahapatra, 1985), two different pulse spacings, T_1 and T_2, are used alternately (Fig. 2.1a). Then, alternate pairs of return samples are used to compute autocorrelation estimates, R_1 at lag T_1 and R_2 at lag T_2. The velocity is estimated from the phase difference between the two using the formula,

$$\hat{v} = \lambda \; arg(R_1 R_2^*) \; / \; [4\pi(T_2 - T_1)] \; . \qquad (2.1)$$

Thus, the difference in PRT, $(T_2\text{-}T_1)$, determines the unambiguous velocity, v_a, for the staggered PRT technique and is given by

$$v_a = \pm \lambda \,/\, [4(T_2\text{-}T_1)] \;;\;\; T_1 < T_2 \,. \tag{2.2}$$

Zrnic and Mahapatra (1985) suggest a testing procedure to estimate mean velocity and signal power for echoes received within the time delay (T_1+T_2). In theory, this seems to be possible because the overlaid signals in any two consecutive samples are from two different ranges and, therefore, are uncorrelated. Thus, the expected value of the overlaid signal contribution to the autocorrelation is zero, and the effective unambiguous range becomes

$$r_a = c(T_1+T_2)/2. \tag{2.3}$$

Eq. 2.1 and 2.3 suggest that the staggered PRT is equivalent to a uniform PRT = (T_1+T_2) for the unambiguous range and a uniform PRT = $(T_2\text{-}T_1)$ for the unambiguous velocity, and each can be selected independently. However, the practical utility of this scheme is limited due to the quality of estimates. The overlaid signal increases the variance of the estimates because it acts as noise. Thus, the ratio of the overlaid signal powers is the equivalent signal-to-noise ratio (SNR), and for a reasonable accuracy of the estimates, the unwanted signal has to be at least 3 dB below the desired signal power.

Let $r_{a1} = cT_1/2$ and $r_{a2} = cT_2/2$ so that $r_a = r_{a1} + r_{a2}$. If r_{a1} is chosen sufficiently large so that no echoes are received from ranges greater than r_{a1}, then the problem of overlaid echoes could be eliminated. For weather radars, r_{a1} would have to be 460 km (for 0.5 deg. elevation scan), but this would degrade the variance of the estimates considerably. Thus, the practical limit for r_{a1} is smaller than 460 km unless some means of separating the overlaid signals is employed. It may be possible to extend the unambiguous range to r_{a2} with some additional processing to resolve the resulting single signal overlay (i.e., alternate samples only have overlaid echoes) in some of the range gates. This possibility will be explored in the future, but in this report, we consider data which have no overlay.

It is shown by Zrnic and Mahapatra (1985) that the standard error in the velocity estimate increases as the ratio $\kappa = T_1/T_2$ approaches unity, and a good choice is κ =2/3. Thus, the unambiguous range and unambiguous velocity are indirectly tied in practice via the estimate accuracy. However, compared to the uniform PRT, it is possible to achieve a much larger r_a and v_a because the limiting equation is $v_a\, r_{a1} = [\kappa/(1-\kappa)\}c\lambda/8$ for the staggered PRT scheme.

A major problem with staggered PRT scheme has been the ground clutter filtering. The non-uniform sampling aliases power from certain Doppler frequencies into the clutter frequency band around zero Doppler. Then, filtering the clutter also removes the aliased signal power from a band of spectral coefficients and introduces phase perturbations at these bands which bias the velocity estimate. The widths of these bands depend on the spectrum width of the signal as well as the clutter filter width. Banjanin and Zrnic (1991) have investigated several methods of ground clutter filtering to mitigate the phase perturbations. A scheme they proposed uses two filters sequentially such that the overall filter coefficients are time varying. In the Doppler bands where the filter phase response is not linear, special decision logic corrects velocity estimates. To overcome these obstacles, Chornoboy (1993) proposed a processing technique applied to a block staggered sampling and a least squares design of a filter matrix to achieve a desired frequency response. The added complexity of the pulse pattern enables an improved balance between the magnitude and the phase response so that Chornoboy(1993) achieved satisfactory results.

In the following sections, we present a different and novel approach to the clutter filtering and spectral moment estimation for the staggered PRT sequence.

3. A new approach to processing the staggered PRT samples:

In the proposed new approach, we seek to reconstruct the spectrum of the weather signal from the staggered time series samples, i.e., reconstruct the spectrum of the time series with a uniform sampling period of T_u, starting from the staggered PRT samples sequence, and then estimate the spectral moments from this reconstructed spectrum. This procedure allows estimation of the spectral moments with a much lower variance than the earlier methods. Further, a novel method of clutter filtering in the spectral domain has been proposed which can achieve a clutter suppression in excess of 40 dB and near complete elimination of all the spurious rejection bands in the $\pm v_a$ interval encountered by other methods of clutter filtering. This is the most important feature of the clutter suppression scheme which makes it practical to use the staggered PRT scheme in weather radars. The processing procedure has two major parts, 1) the reconstruction of an equivalent uniform PRT sequence for spectral moment estimation, and 2) the clutter filtering and residual bias removal. These two are interleaved because the clutter has to be filtered before the spectral moments are estimated. We will explain the reconstruction of the equivalent uniform PRT sequence first however, and then proceed to explain the clutter filtering procedure. These two procedures are incorporated in the algorithm presented later in this report.

3.1. Reconstruction of the signal spectrum.

Our technique requires a small restriction on the selection of the two PRTs used in the staggered PRT transmission. If T_1 and T_2 are the PRTs, we select them such that they are integer multiples of some basic PRT T_u, so that $T_1=n_1T_u$, and $T_2=n_2T_u$, where n_1 and n_2 are integers. Although n_1 and n_2 can be any integers in general, a good choice is $n_2=n_1+1$ from the point of the performance of the staggered PRT scheme. Thus, $(T_2-T_1)=(n_2-n_1)T_u$ determines the unambiguous velocity, v_a, and the unambiguous range, $r_{a1}=cT_1/2$. This, of course, assumes that T_1 is chosen sufficiently large so that no second trip overlay occurs.

Let g_i, i=1,2,3,...M, (M even) be the samples of the weather signal sampled at time intervals T_1 and T_2, alternately. We introduce zeros in g_i to form a sample sequence v_i of length $N=(n_1+n_2)M/2$ with a uniform sampling period of T_u, in which the missing samples are represented by zeros (see Fig. 2.1b). We call this the derived time series. Let c_i be a code sequence of length N obtained by replacing all the g_i samples in v_i by unity. For example, c_i =[1010010100... etc.] for $\kappa = T_1/T_2 = n_1/n_2 = 2/3$. We can write the sample sequence v_i as a product of the sequence c_i and e_i, where e_i is the signal time series sampled at T_u intervals.

$$v_i = c_i\, e_i\ ; \qquad i=1,2,3,\dots N. \tag{3.1}$$

Having converted the staggered PRT sequence into a uniformly sampled sequence (with missing samples represented by zeros), we can examine the spectrum of the uniform sequence, e_i. Thus, the spectrum of v_i can be represented as a convolution of the spectrum of the code c_i and the spectrum of the complete but unknown signal e_i.

$$\mathrm{DFT}(\ v_i\ ) = \{\ \mathrm{DFT}(\ c_i\ ) \star \mathrm{DFT}(\ e_i\ )\ \} \tag{3.2}$$

where $\star$ represents circular convolution, and DFT( ) represents the discrete Fourier transform of the sequence in brackets. We use the capital letters to denote the spectral coefficients of the corresponding time domain quantities denoted by lowercase letters and capital bold face letters to denote matrices. Subscript index '*i*' is used for the time domain quantities, and subscript index '*k*' is used for the spectral coefficients. For example, E_k = DFT(e_i), are the spectral coefficients, and **E** is the column matrix of coefficients E_k . Eq. (3.2) can be written in matrix form as

$$\mathbf{V} = \mathbf{C}\ \mathbf{E}. \tag{3.3}$$

V and **E** are (Nx1) column matrices containing the spectral coefficients, V_k and E_k, of the corresponding time sequences, v_i and e_i, and **C** is the convolution matrix (size: NxN) whose column vectors are cyclically shifted versions of C_k . To preserve the power in the spectrum, the convolution matrix, **C**, is normalized such that each column vector is a unit vector (i.e., the norm of each column vector is unity). Note that by normalizing the column vectors, the row vectors are also normalized automatically. Our objective here is to reconstruct the spectrum E_k from the samples g_i. It is observed that the convolution matrix is singular (rank of **C** equals the number of staggered PRT samples, M), and hence, it cannot be inverted to get E_k. If we discard the phases of the convolution matrix elements, the matrix becomes non-singular. It may be noted that the magnitude of the spectral coefficients $abs\{E_k\}$ is sufficient to compute the autocorrelation $R(T_u)$, and hence, the velocity and spectrum width. Therefore, we attempt to retrieve $abs(\mathbf{E})$ using $abs(\mathbf{C})$, which can be inverted.

In general, $abs\{\mathbf{CE}\} \neq abs\{\mathbf{C}\}abs\{\mathbf{E}\}$ because of the complex addition of the coefficients in the process of matrix multiplication. However, because the convolution matrix, **C**, has only (n_1+n_2) non-zero coefficients separated by $N/(n_1+n_2)$ coefficients for $\kappa = n_1/n_2$ in each row (or column), we can show that the complex addition does not take place in the process of convolution if the signal spectrum is narrow so that its spread is less than $N/(n_1+n_2)$ coefficients, and hence, the equality is valid. Then, the convolved matrix **V** has (n_1+n_2) spectral replicas of the original spectrum, **E** (in the complex domain, each replica has a specific complex multiplier), and these replicas do not overlap. Thus, we define a "magnitude deconvolution" as

$$abs\{\mathbf{E}\} = [\ abs\{\mathbf{C}\}]^{-1}\ abs\{\mathbf{V}\}, \qquad (3.4)$$

where $[\ abs\{\mathbf{C}\}]^{-1}$ is the magnitude deconvolution matrix. It is important to note that this equation provides the exact magnitude spectrum only under the condition that the non-zero spectral coefficients of the signal e_i spread at most $N/(n_1+n_2)$ coefficients or that the total spread must be less than

$$2v_a/(n_1+n_2) \qquad (3.5)$$

in the velocity domain. This condition means the spectrum cannot alias on itself and is obtained from the average sampling rate $2/(T_1+T_2)$ for the staggered sequence. Although wider spectra are not reproduced with high fidelity using this procedure (because of the overlap of spectral coefficients in the spectrum V_k), the non-ideal reconstruction of wider spectra does not generally bias the velocity estimate but affects its variance. If T_1 and T_2 are judiciously chosen, the criterion (Eq. 3.5) can be nearly satisfied for most weather signals. For example, at a 10 cm wavelength and $v_a = \pm 50$ m s^{-1}, Eq. 3.5 can be nearly satisfied for width, $w<6$ m s^{-1}. Nonetheless, it is important to note that the criteria is not satisfied exactly even for $w=4$ m s^{-1}.

Most of the results presented in this report are based on simulation studies. All the computations were carried out using simulated staggered PRT time series. The staggered PRT