

# Signal Design and Processing Techniques for WSR-88D Ambiguity Resolution

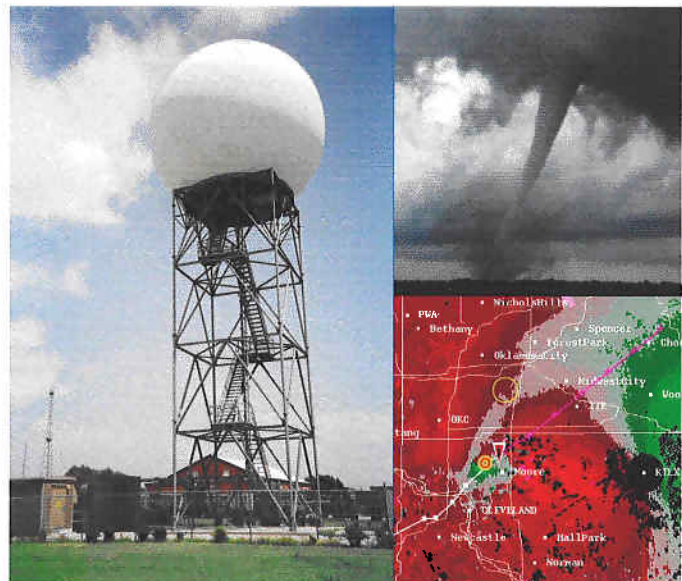
## Staggered PRT Technique

National Severe Storms Laboratory Report

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*with contributions by:* D. S. Zrnic´ and R. J. Doviak

**Part 4**  
**October 2000**



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**SIGNAL DESIGN AND PROCESSING TECHNIQUES  
FOR WSR-88D AMBIGUITY RESOLUTION**

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Part-4: staggered PRT**

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# SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

## Part - 4: the staggered PRT technique

### 1. Introduction

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL) to address the mitigation of range and velocity ambiguities in the WSR-88D. This is the fourth report in the series that deals with velocity and range ambiguity resolution in the WSR-88D. The first two reports mainly dealt with uniform PRT transmission and phase coding techniques to resolve the range ambiguity. Although the phase coding techniques do not directly address the velocity ambiguity problem, their capability to separate overlaid echoes allows the use of shorter PRTs which, in turn, diminishes the occurrence of ambiguous velocities. In the third part, we considered the staggered PRT technique and its variants. The significant results in the Report 3 are a new staggered PRT sequence processing scheme in the spectral domain which significantly improves spectral moment estimates, and a clutter filtering method that recovers velocity information over the entire extended unambiguous velocity interval without any drop-out regions. The only assumption made in the algorithm is that there is no overlaid signal. This necessarily restricts the selection of the PRT  $T_I$  to be sufficiently large for a given elevation so that the probability of overlay is small.

After the third report was submitted in July 1999, some more ideas were explored in an effort to further improve the staggered PRT scheme. Specifically, we tried to further improve the velocity estimate errors by optimizing the window weights. We also examined the possibility of increasing the unambiguous range to  $r_{a2}$ , by resolving the overlaid signal from corresponding to the shorter range  $r_{a1}$ . Exhaustive simulations were carried out to evaluate the performance of the staggered PRT decoding scheme and determine the limits of

spectral moment recovery within acceptable bounds under various conditions. This information is very useful in developing a data censoring strategy to discard or flag the bad data. The results from all these studies has been reported in this Part-4 of the report. In the light of these new results and an enhanced capability of the staggered PRT algorithm, we have revised the proposed WSR-88D scan strategy given in Table.5.3 of Report 3. The notations used in this report are the same as in Report 3. This study is essentially a continuation of the staggered PRT work. A brief introduction to the staggered PRT technique was given in Report 3, and a part of it is repeated here for the convenience of readers and to recall the symbols and notations used in the context of the staggered PRT processing.

## 2. The staggered PRT technique

Here, we describe the staggered PRT scheme briefly before we embark on a discussion of the new method of processing. In the staggered PRT technique (Zrníc and Mahapatra, 1985), two different pulse spacings,  $T_1$  and  $T_2$ ; ( $T_2 > T_1$ ), are used alternately (Report 3, Fig. 2.1a). Then, alternate pairs of return samples are used to compute autocorrelation estimates,  $R_1$  at lag  $T_1$  and  $R_2$  at lag  $T_2$ . The velocity is estimated from the phase difference between the two using the formula,

$$\hat{v} = \lambda \arg(R_1 R_2^*) / [4\pi(T_2 - T_1)] . \quad (2.1)$$

Thus, the difference in PRT, ( $T_2 - T_1$ ), determines the unambiguous velocity,  $v_a$ , for the staggered PRT technique and is given by

$$v_a = \pm \lambda / [4(T_2 - T_1)] ; T_1 < T_2 . \quad (2.2)$$

Zrníc and Mahapatra (1985) suggest a procedure to estimate mean velocity and signal power for echoes received within the time delay ( $T_1 + T_2$ ). In theory, this seems to be possible because the overlaid signals in any two consecutive samples are from two different

ranges and, therefore, are uncorrelated. Thus, the expected value of the overlaid signal contribution to the autocorrelation is zero, and the effective unambiguous range becomes

$$r_a = c(T_1+T_2)/2. \quad (2.3)$$

Eq. 2.1 and 2.3 suggest that the staggered PRT is equivalent to a uniform PRT  $= (T_1+T_2)$  for the unambiguous range and a uniform PRT,  $T_u = (T_2-T_1)$  for the unambiguous velocity, and each can be selected independently. However, the practical utility of this scheme is limited due to the quality of estimates. The overlaid signal increases the variance of the estimates because it acts as noise. Thus, the ratio of the overlaid signal powers is the equivalent signal-to-noise ratio (SNR), and for a reasonable accuracy of the estimates, the unwanted signal has to be at least 3 dB below the desired signal power.

Let  $r_{a1} = cT_1/2$  and  $r_{a2} = cT_2/2$  so that  $r_a=r_{a1}+r_{a2}$ , and  $r_{a1} < r_{a2}$ . If  $r_{a1}$  is chosen sufficiently large so that no echoes are received from ranges greater than  $r_{a1}$ , then the problem of overlaid echoes could be eliminated. If we choose PRTs such that we have no echoes from range greater than  $r_{a2}$  then for some of the range gates, the alternate samples have overlaid signal from another range gate separated by a delay time  $T_1$ , and the rest are free of overlaid signal. This situation, we call the “one-overlay” situation, is thought to be a good candidate for further extending the unambiguous range to  $r_{a2}$ , with some additional processing. This possibility has been explored in this report.

It is shown by Zrnic and Mahapatra (1985) that the standard error in the velocity estimate increases as the ratio  $\kappa = T_1/T_2$  approaches unity, and a good choice is  $\kappa=2/3$ . Thus, the unambiguous range and unambiguous velocity are indirectly tied in practice via the estimate accuracy. However, compared to the uniform PRT, it is possible to achieve a much larger  $r_a$  and  $v_a$  because the limiting equation is  $v_a r_{a2} = \{1/(1-\kappa)\}c\lambda/8$  for the staggered PRT scheme with one-overlay resolution. Report 3 indicates that  $\kappa=2/3$  is optimum irrespective of the decoding algorithm, hence in this study we examine the  $\kappa=2/3$  case only. Some discussion on other values of  $\kappa$  is available in Report 3.

### 3. Review of the staggered PRT sequence processing

The estimation of the spectral moments of the weather echo from the staggered PRT sequence is based on a few key ideas. The first idea is to view the non-uniform sample sequence as a product of a uniform sample sequence and a binary code sequence. Thus, the spectrum of the staggered PRT sequence can be viewed as a convolution in the spectral domain. Because of the singular nature of the convolution matrix, the de-convolution cannot be carried out. However, under certain conditions of narrow weather spectra, “magnitude de-convolution” can recover the spectral magnitudes, but not the complex coefficients (Sachidananda and Zrnic 2000). The phases are not required for estimating the spectral moments. Thus, the velocity estimation is identical to that of the pulse pair processing, except that the autocorrelation is not computed using the pairs of pulses but from the power spectrum. One of the major advantages of this procedure is that the standard errors in the velocity estimates are much lower than errors in estimates obtained by the conventional pulse pair processing. This comes about because, in the pulse pair processing of staggered PRT data (Zrnic and Mahapatra 1985), autocorrelations are computed at two different lags,  $T_1$  and  $T_2$ , and the phase difference is used for computing the velocity, whereas in the new approach the autocorrelation for lag  $(T_2 - T_1)$  is directly computed. Incidentally, this is a spin-off from the approach that we adopted to filter the clutter from the staggered PRT sequence. When the staggered PRT work was undertaken a method for clutter filtering was sought, because that was one of the major hurdles which prevented implementation of the staggered PRT technique in operational radars.

The second idea, central to the filtering of the clutter from the staggered PRT sequence, is a technique to recover the spectral coefficients of the weather signal in the region where the clutter and signal are overlapping in the spectral domain. Any conventional filtering technique cannot distinguish between the signal and the clutter power in a given spectral coefficient, and filtering one would automatically eliminate the other too. In this new procedure, the modulation properties of the code sequence, under the narrow spectra condition, are utilized to retain some fraction of the signal power while filtering the clutter power completely. An estimate of the complex clutter spectral coefficient is first obtained by

projecting the complex spectral code vector (appropriate column vector of the convolution matrix) onto a set of spectral coefficients where clutter is expected (5 coefficients for  $\kappa=2/3$ ), and this estimated clutter vector is subtracted from the set of coefficients. Because of the linear independence between the different spectral code vectors, not all the weather signal power is filtered, except when the weather signal velocity also is close to zero (ground clutter is always around zero Doppler), and this residual signal power (and some additional information) is used to restore the signal to its original value. This is possible, because the residual power left in the spectral coefficient after the clutter is filtered is a known fraction of the original signal power. The correction factor can be easily computed provided the location of the original signal component can be determined. This is accomplished by obtaining an approximate velocity estimate with the partial signal power. The procedure fails only when the clutter filter is very wide and the approximate initial velocity estimate is not within  $\pm v_u/5$  of the actual value. Without this last step, which is termed the “bias removal procedure”, the velocity estimate is slightly biased because of the loss of signal power from the region of the spectrum from where the clutter is filtered. This bias becomes significant for larger clutter filter widths. The upper limit for the number of coefficients from which clutter can be filtered is a little more than half the number of the total spectral coefficients, and is generally sufficient to recover velocity in the presence of clutter as large as 50dB more than the signal power. If the clutter-to-signal ratio (CSR) is lower, a narrower clutter filter would be sufficient, and for sufficiently narrow clutter filter widths (e.g.,  $n_c < 7$  for a number of  $T_u$  intervals  $N=160$ , and a number of staggered samples  $M=64$ ), the bias correction may not even be necessary (for CSR less than about 15 dB). A very brief account of the spectrum reconstruction and the clutter filtering procedure is given below in the next two sub-sections to provide continuity for the reader.

### ***3.1. Reconstruction of the signal spectrum***

As indicated earlier in the introduction, unless indicated otherwise the stagger ratio,  $\kappa=2/3$ , is assumed in all our discussions and results in this report. If  $T_1$  and  $T_2$  are the PRTs, we select  $T_1=2T_u$ , and  $T_2=3T_u$ . Therefore, a uniform sample sequence can be

constructed for the sampling period  $T_u$ ; then the corresponding code is 10100... etc., with ones representing the samples available, and the zeros, the missing samples. If the first sample is from the  $T_2$  pulse transmission the code would be 10010... etc. If  $e_i$  is the uniform sample sequence and  $c_i$  is the code sequence, then the available stagger PRT sample sequence,  $v_i$ , can be written as

$$v_i = c_i e_i ; \quad i=1,2,3,\dots N , \quad (3.1)$$

and in the transform domain this is a convolution represented by

$$\text{DFT}( v_i ) = \{ \text{DFT}( c_i ) \star \text{DFT}( e_i ) \} \quad (3.2)$$

where the  $\star$  represents circular convolution, and the  $\text{DFT}( )$  represents the discrete Fourier transform of the sequence in brackets.  $N$  is the number of samples after inserting zeros for the missing samples. If  $M$  is the number of staggered PRT samples, then  $N=5M/2$  for  $\kappa=2/3$ . We use capital letters to denote the spectral coefficients, and the corresponding time domain quantities are denoted by lowercase letters. The bold face letters denote matrices. Subscript index 'i' is used for the time domain quantities, and subscript index 'k' is used for the spectral coefficients. For example,  $E_k = \text{DFT}(e_i)$ , are the spectral coefficients, and  $\mathbf{E}$  is the column matrix of coefficients  $E_k$ . Eq. (3.2) can be written in matrix form as

$$\mathbf{V} = \mathbf{C} \mathbf{E}. \quad (3.3)$$

$\mathbf{V}$  and  $\mathbf{E}$  are  $(N \times 1)$  column matrices containing the spectral coefficients,  $V_k$  and  $E_k$ , of the corresponding time sequences,  $v_i$  and  $e_i$ , and  $\mathbf{C}$  is the convolution matrix (size:  $N \times N$ ) whose column vectors are cyclically shifted versions of  $C_k$ . Because the convolution matrix is singular, we recover the magnitude spectrum using the magnitude deconvolution defined by

$$\text{abs}\{\mathbf{E}\} = [ \text{abs}\{\mathbf{C}\} ]^{-1} \text{abs}\{\mathbf{V}\}, \quad (3.4)$$

where  $[abs\{\mathbf{C}\}]^{-1}$  is the magnitude deconvolution matrix. This reconstructs the exact magnitude spectrum only under the condition of “narrow” spectra; that is, the non-zero spectral coefficients of the signal  $e_i$  are spread at most  $N/5$  coefficients, or the total spread is less than  $2v_u/5$  for  $\kappa=2/3$  (see Report 3 or Sachidananda and Zrnic 2000).

Once the magnitude spectrum is obtained, the spectral domain equivalent of the pulse pair algorithm can be used to estimate the mean power,  $p$ , mean velocity,  $v$ , and spectrum width,  $w$ . If the ground clutter is present, the echo time series is filtered using the procedure described briefly in the next sub-section, and then the magnitude deconvolution is applied.

### 3.2. Ground clutter filtering

The clutter filter capitalizes on the cyclic property of the convolution matrix and the assumption that the weather signal spectra are “narrow” (as defined earlier). The code spectrum has only 5 non-zero coefficients spaced 1/5th of the total (unambiguous velocity) span, hence, only 5 spectral coefficients will be involved at a time in the convolution process. Specifically, the power in each spectral coefficient is spread over these 5 coefficients, hence the problem of clutter filtering or the spectrum reconstruction can be split into  $N/5$  equations with 5 variables each, in place of one equation with  $N$  variables. For example the equation

$$\mathbf{V}_r = \mathbf{C}_r \mathbf{E}_r, \quad (3.5)$$

(see Report 3, Eq 3.7 for details) consisting of the rearranged matrices is a representation of  $N/5$  equations together in matrix form. To understand the clutter filtering procedure it is sufficient to consider one such equation, say the  $k^{th}$  column of the matrices  $\mathbf{E}_r$  and  $\mathbf{V}_r$ , and form a 5x5 matrix equation.

Consider an example with parameters  $\kappa=2/3$ ,  $M=64$  for which  $N=160$ , and let  $k=1$ . The corresponding equation is

$$\begin{bmatrix} V_1 \\ V_{33} \\ V_{65} \\ V_{97} \\ V_{129} \end{bmatrix} = \begin{bmatrix} C_1 & C_{129} & C_{97} & C_{65} & C_{33} \\ C_{33} & C_1 & C_{129} & C_{97} & C_{65} \\ C_{65} & C_{33} & C_1 & C_{129} & C_{97} \\ C_{97} & C_{65} & C_{33} & C_1 & C_{129} \\ C_{129} & C_{97} & C_{65} & C_{33} & C_1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_{33} \\ E_{65} \\ E_{97} \\ E_{129} \end{bmatrix}. \quad (3.6)$$

The left hand side represents the convolved spectral coefficients (available from the measurement), and the coefficients  $E_k$  are for a uniform sequence of the signal plus the clutter. Under the condition of “narrow” weather spectra, only one of its five ( $E_1$  or  $E_{33}$  or  $E_{65}$  or  $E_{97}$  or  $E_{129}$ ) coefficients is non zero; the clutter can be present in the first coefficient (or in the last coefficient for a different  $k$ ), because the clutter is at zero Doppler. The amplitude and the position of the signal coefficient that we are trying to recover is not known. Then it can be shown that the column vector,  $\mathbf{V}$ , on the left is  $E_l$  times the first column vector of the convolution matrix,  $\mathbf{C}_r$ , plus the signal spectral coefficient (whichever is non zero) multiplied by the corresponding column vector of  $\mathbf{C}_r$ . Therefore, if we estimate the complex amplitude of the first column vector of  $\mathbf{C}_r$  present in  $\mathbf{V}$  and subtract it, all the clutter is removed. However, because of the non-orthogonality of the columns of  $\mathbf{C}_r$ , part of the signal power is also removed (signal is completely removed when it is also in the first coefficient, or at the zero Doppler). That is, the projection of the signal vector on to the clutter vector is also removed. The remaining part of the signal has certain relationship to the original signal and hence the original signal can be recovered using the bias removal procedure (see Report 3 for details).

