STAGGERED PRT

ALGORITHM DESCRIPTION

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ASSUMPTIONS

1) The transmission sequence alternates two pulse repetition times (PRT) as: T_1, T_2, T_1, T_2, … for a total of M pulses.

2) The PRT ratio $T_1/T_2 = \kappa_m/\kappa_n$ is larger than 1/3, where κ_m and κ_n are relatively prime integers.

3) All range gates are available and there is a perfect alignment of range gates between the two PRTs (i.e., a given range gate represents the same resolution volume in space for every transmitted pulse). Also, the number of range gates for each PRT is: $N_1 = T_1/\tau_s$ and $N_2 = T_2/\tau_s$, where τ_s is the sampling period.

4) There are no significant echoes beyond $\max(r_{a1}, r_{a2})$, where r_{ai} is the maximum unambiguous range corresponding to T_i

5) It is *not* assumed that M is even or that $T_1 < T_2$.

INPUTS

1) Complex time-series data:

$V(n,m) = I(n,m) + jQ(n,m)$, where $0 \leq n < N_1$ for even m, $0 \leq n < N_2$ for odd m, and $0 \leq m < M$. Note that n indexes the range gates and m the sweeps (or pulses).

2) Associated metadata:

N is the noise power in linear units
SYSCAL is the system calibration constant in dB
ATMOS is the elevation-dependent atmospheric attenuation in dB/km

3) Ground clutter filter bypass map:

$B(n)$, where n indexes the range bins with the same resolution as the time-series data along a radial, and the map corresponds to the elevation and azimuth of the radial being processed. B is 0 if clutter filtering is required and 1 otherwise.

OUTPUTS

1) Reflectivities, Doppler velocities, and spectrum widths:

$Z(n)$	for $0 \leq n < \max(N_1,N_2)$,
$v(n)$ and $w(n)$	for $0 \leq n < \max(N_1,N_2)$.

2) Signal-to-noise ratio and overlaid censoring flags:

$NS_Z(n)$, $NS_V(n)$ and $NS_W(n)$	for $0 \leq n < \max(N_1,N_2)$,
$OV(n)$	for $0 \leq n < \max(N_1,N_2)$.

HIGH-LEVEL ALGORITHM DESCRIPTION

If the PRT ratio has changed
1. Pre-computation of velocity de-aliasing rules

End
For each range bin n, where $0 \leq n < \max(N_1,N_2)$
2. Clutter filtering
3. Power and correlation computations for each PRT

End
4. Short/long PRT data swap

For each range bin n, where $0 \leq n < N_2$
5. Combined power computation

End
6. Strong point clutter canceling

For each range bin n, where $0 \leq n < N_2$
7. Signal power computation
8. Reflectivity computation
9. Velocity computation
10. Spectrum width computation
11. Determination of significant returns for reflectivity
12. Determination of significant returns for velocity
13. Determination of significant returns for spectrum width
14. Determination of overlaid returns for velocity and spectrum width

End

STEP-BY-STEP ALGORITHM DESCRIPTION

1) Pre-computation of velocity de-aliasing rules

This method is described in the paper "Design, Implementation, and Demonstration of a Staggered PRT Algorithm for the WSR-88D" by Torres et al. (2004). Herein, VDA_c are the normalized velocity difference transfer function (VDTF) constant values and VDA_p are the normalized number of Nyquist co-intervals for de-aliasing.

A set of velocity de-aliasing rules could be pre-computed for each new PRT ratio as follows:

(Compute type-I and II positive (VDTF) discontinuity points. κ_m and κ_n are the integers in the PRT ratio)
$p = 0$
While $2p + 1 < \kappa_m$
    $D_1(p) = (2p + 1)/\kappa_m$
    $TYPE_1(p) = 1$
    $p = p + 1$
End
$q = 0$
While $2q + 1 < \kappa_n$
    $D_2(q) = (2q + 1)/\kappa_n$
    $TYPE_2(q) = 2$
    $q = q + 1$
End
(Create TYPE by combining and sorting and both sets of discontinuity points)
Concatenate D_1 and D_2 to create D with $p + q$ elements.
Concatenate $TYPE_1$ and $TYPE_2$ to create $TYPE$ with $p + q$ elements.
Sort $TYPE$ in a "slave" mode using D as the "master".

(Compute VDTF constants and de-aliasing factors for non-negative discontinuity points)

$VDA_c(p+q) = 0$
$VDA_p(p+q) = 0$
For $0 \le k < p+q$
    If $TYPE(k) = 1$
        $VDA_c(p+q+k+1) = VDA_c(p+q+k) - 2/\kappa_m$
        $VDA_p(p+q+k+1) = VDA_p(p+q+k) + 1/\kappa_m$
    Else
        $VDA_c(p+q+k+1) = VDA_c(p+q+k) + 2/\kappa_n$
        $VDA_p(p+q+k+1) = VDA_p(p+q+k)$
    End
End

(Compute VDTF constants and de-aliasing factors for negative discontinuity points)

For $-(p+q) \le k < 0$
    $VDA_c(p+q+k) = -VDA_c(p+q-k)$
    $VDA_p(p+q+k) = -VDA_p(p+q-k)$
End

PROCESSING STEPS

2) Clutter filtering

The clutter filtering algorithm removes the mean (or DC) component of V in those locations where the site-dependent clutter filter bypass map B indicates the need for clutter filtering (here, it is assumed that B corresponds to the azimuth and elevation of the current time-series data). V_m is the DC component of V computed using all sweeps where available, and only long-PRT sweeps beyond the short PRT.

If $B(n) = 0$
    (Clutter filtering is required)
    $V_{sum} = 0$
    $K = 0$
    If $n < N_1$
        (Accumulate even pulses, if available)

$$V_{sum} = V_{sum} + \sum_{m=0}^{K_s^{(1)}-1} V(n, 2m)$$

$$K = K + K_s^{(1)}$$

    End
    If $n < N_2$
        (Accumulate odd pulses, if available)

$$V_{sum} = V_{sum} + \sum_{m=0}^{K_s^{(2)}-1} V(n, 2m+1)$$

$$K = K + K_s^{(2)}$$

    End
    (Compute mean using total number of pulses accumulated)
    $V_m = V_{sum} / K$
Else
    (Clutter filtering is not required)
    $V_m = 0$
End

(Apply ground clutter filtering, if needed)

If $n < N_1$
    (Subtract mean from even pulses, if available)

For $0 \le m < K_s^{(1)}$

$V_F(n, 2m) = V(n, 2m) - V_m$

End

End

If $n < N_2$

(Subtract mean from odd pulses, if available)

For $0 \le m < K_s^{(2)}$

$V_F(n, 2m+1) = V(n, 2m+1) - V_m$

End

End

3) Power and correlation computations for each PRT

If $n < N_1$

(Compute power from even pulses, if available)

$$P_1(n) = \frac{1}{K_s^{(1)}} \sum_{m=0}^{K_s^{(1)}-1} \left| V_F(n, 2m) \right|^2$$

End

If $n < N_2$

(Compute power from odd pulses, if available)

$$P_2(n) = \frac{1}{K_s^{(2)}} \sum_{m=0}^{K_s^{(2)}-1} \left| V_F(n, 2m+1) \right|^2$$

End

If $n < \min(N_1, N_2)$

(Compute lag-1correlations from all pulses, if available)

$$R_1(n) = \frac{1}{K_p^{(1)}} \sum_{m=0}^{K_p^{(1)}-1} V_F^*(n, 2m) V_F(n, 2m+1)$$

$$R_2(n) = \frac{1}{K_p^{(2)}} \sum_{m=0}^{K_p^{(2)}-1} V_F^*(n, 2m+1) V_F(n, 2m+2)$$

End

In the previous equations, K_s is the number of sweeps (pulses) used in the power computations, and K_p is the number of pairs used in the lag-1 correlation computations. These constants depend on the total number of sweeps M, and they may differ for short and long PRT estimates depending on the parity of M as

$$K_s^{(1)} = \begin{cases} \frac{M}{2} & \text{if } M \text{ is even} \\ \frac{M+1}{2} & \text{if } M \text{ is odd} \end{cases}, \qquad K_s^{(2)} = \begin{cases} \frac{M}{2} & \text{if } M \text{ is even} \\ \frac{M-1}{2} & \text{if } M \text{ is odd} \end{cases},$$

$$K_p^{(1)} = \begin{cases} \frac{M}{2} & \text{if } M \text{ is even} \\ \frac{M-1}{2} & \text{if } M \text{ is odd} \end{cases}, \text{ and } \quad K_p^{(2)} = \begin{cases} \frac{M-2}{2} & \text{if } M \text{ is even} \\ \frac{M-1}{2} & \text{if } M \text{ is odd} \end{cases}.$$

4) Short/long PRT data swap

This step is done to simplify the logic of the algorithm by making all variables with subscript 1 correspond to the short PRT, and variables with subscript 2 correspond to the long PRT.

If $T_2 < T_1$

Swap P_1, R_1, T_1, and N_1 with P_2, R_2, T_2, and N_2 , respectively

End

5) Combined power computation

To use as much information as possible, data are extracted from the two power arrays with different rules for each of the three segments depicted in Figure 1. For segment I, data are extracted only from P_1, since P_2 may be contaminated on those range bins with overlaid powers. An average of P_1 and P_2 is extracted for segment II, given that both power vectors are "clean" there. Finally, segment III data are obtained from P_2. In algorithmic form,

If $n < \min(N_1, N_2 - N_1)$
    (Segment I)
    $P(n) = P_1(n)$
ElseIf $n < N_1$
    (Segment II)
    $P(n) = \frac{1}{2}\left[P_1(n) + P_2(n)\right]$
Else
    (Segment III)
    $P(n) = P_2(n)$
End

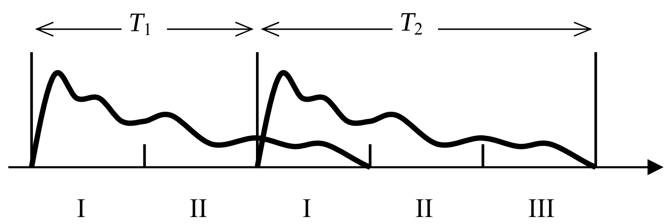

Fig. 1. Signal powers in the staggered PRT algorithm. Roman numerals indicate segment numbers.

6) Strong point clutter canceling

Processing is as in the current system. Strong-point clutter canceling is applied to P, P_1, P_2, R_1, and R_2 based on P powers.

7) Signal power computation

If $P(n) < N$
    $S = 0$
Else
    $S = P(n) - N$
End

N is the noise power in linear units

8) Reflectivity computation

(Range in km)
$R = n\Delta R + \Delta R/2$
(Reflectivity in dBZ)
$Z(n) = 10\log_{10}(S) + SYSCAL + R\ ATMOS + 20\log_{10}(R)$,

where ΔR is the spacing between range gates in km ($\Delta R = c\tau_s/2$), *SYSCAL* is the system calibration constant in dB, and *ATMOS* is the atmospheric attenuation in dB/km depending on the antenna elevation angle.

9) Velocity computation

If $n < N_1$

(Compute Doppler velocities for each PRT using the corresponding correlation estimates)

$$v_1 = -\frac{\lambda}{4\pi T_1}\arg\left[R_1(n)\right]$$

$$v_2 = -\frac{\lambda}{4\pi T_2}\arg\left[R_2(n)\right]$$

(Compute extended Nyquist velocity)

$$v_a = \frac{\lambda \kappa_m}{4T_1}$$

(De-alias velocity using pre-computed rules)

$$l = \arg\min_k \left|v_1 - v_2 - VDA_c(k)v_a\right|$$

$$v(n) = v_1 + 2v_a \; VDA_p(l)$$

Else

(This value is irrelevant)

$v(n) = 0$

End

10) Spectrum width computation

The spectrum width estimator corresponds to the algorithm implemented in the legacy WSR-88D signal processor. N is the noise power in linear units.

If $n < N_1$

If $S = 0$

(Spectrum width of white noise)

$$w(n) = \frac{\lambda}{4\sqrt{3}T_1}$$

ElseIf $S < \left|R_1(n)\right|$

(Spectrum width of a constant)

$w(n) = 0$

Else

$$w(n) = \frac{\lambda}{2\sqrt{2}\pi T_1}\sqrt{\ln\left(\frac{S}{\left|R_1(n)\right|}\right)}$$

End

Else

(This value is irrelevant)

$w(n) = 0$

End

11) Determination of significant returns for reflectivity

The non-significant return indicator array (NS) is a binary array where 0 indicates “significant” and 1 indicates “non-significant”

If $S < N10^{0.1T_Z}$

$NS_Z(n) = 1$

Else

$NS_Z(n) = 0$

End

T_Z is the reflectivity threshold in dB and N is the noise power in linear units.

12) Determination of significant returns for velocity

The non-significant return indicator array (NS) is a binary array where 0 indicates "significant" and 1 indicates "non-significant"

If $S < N10^{0.1T_V}$
    $NS_V(n) = 1$
Else
    $NS_V(n) = 0$
End

T_V is the velocity threshold in dB and N is the noise power in linear units.

13) Determination of significant returns for spectrum width

The non-significant return indicator array (NS) is a binary array where 0 indicates "significant" and 1 indicates "non-significant"

If $S < N10^{0.1T_W}$
    $NS_W(n) = 1$
Else
    $NS_W(n) = 0$
End

T_W is the spectrum width threshold in dB and N is the noise power in linear units.

14) Determination of overlaid returns for velocity and spectrum width

Censoring of velocity and spectrum width data is only necessary in segment I. This is done by analyzing P_1 in segment I and P_2 in segment III (see Fig. 1). The idea is to determine whether second trip signals mask first trip signals in segment I of P_2. While such overlaid echoes appear in every other pulse and do not bias velocity estimates at those range locations, overlaid powers act as noise. Therefore, when second trip powers in segment I of P_2 are above a preset fraction of their first trip counterparts, the corresponding velocity and spectrum width estimates exhibit very large errors and must be censored. The overlaid indicator array (OV) is a binary array where 0 indicates "not overlaid" and 1 indicates "overlaid". Herein, T_O is the overlaid threshold in dB which is sometimes referred to as TOVER.

If $n < \min(N_1, N_2 - N_1)$
    (Segment I: Range gates that may or may not have overlaid echoes)
    If $P_1(n) > P_2(n + N_1)\ 10^{0.1T_O}$
        $OV(n) = 0$
    Else
        $OV(n) = 1$
    End
ElseIf $n < N_1$
    (Segment II: Range gates that, based on our assumptions, never have overlaid echoes)
    $OV(n) = 0$
Else
    (Segment III: Range gates that are always unrecoverable)
    $OV(n) = 1$
End

Note that when processing the overlaid and significant return flags, the overlaid flags take a lower priority. That is, if a range bin is tagged as non significant and also as overlaid, the overlaid indication is ignored and the gate is treated as a non-significant return only (e.g., painted black as opposed to purple).