Model Verification Using Gaussian Mixture Models

A Parametric, Feature-Based Method

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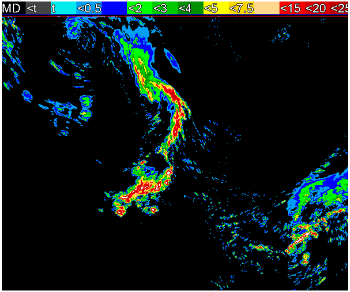
What is a GMM?

Intuitively: find an optimal way to place Gaussian functions at various points in the image such that the sum of these Gaussians mimics the input gridded field.

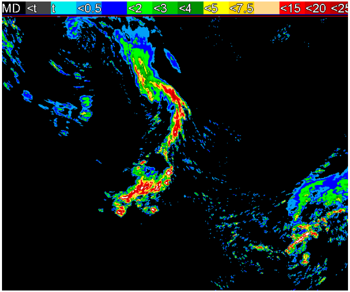


Original

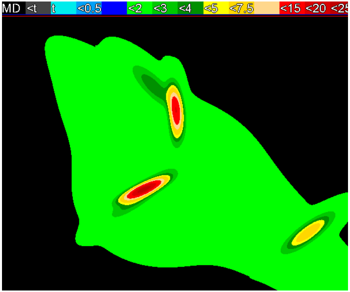
5 Gaussians

Number of Gaussians

The accuracy of fit gets better as you increase number of Gaussians.

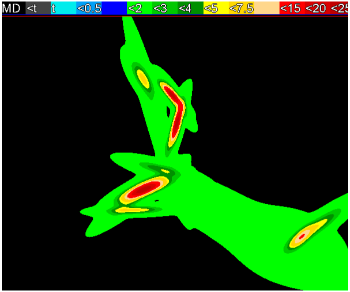


Original



10 Gaussians

Number of Gaussians ...

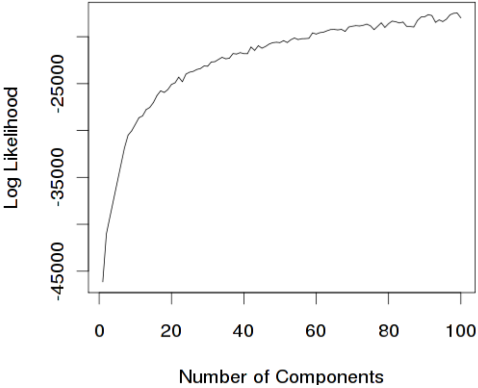


20 Gaussians

50 Gaussians

Diminishing Returns

Keeps getting more and more accurate:



but at some point, the benefits of a parametric model are lost and you might as well just use the pixel values.

The GMM

The GMM is defined as a weighted sum of K two-dimensional Gaussians:

$$G(x,y) = \sum_{k=1}^{K} \pi_k f_k(x,y) \quad (1)$$

$$f(x,y) = \frac{1}{2\pi\sqrt{|\Sigma_{xy}|}} e^{-((x-\mu_x)(y-\mu_y))\Sigma_{xy}^{-1}((x-\mu_x)(y-\mu_y))^T/2} \quad (2)$$

Σ_{xy} is:

$$\begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} \quad (3)$$

Solved using an iterative approach [Lakshmanan and Kain, 2009].

Intensity?

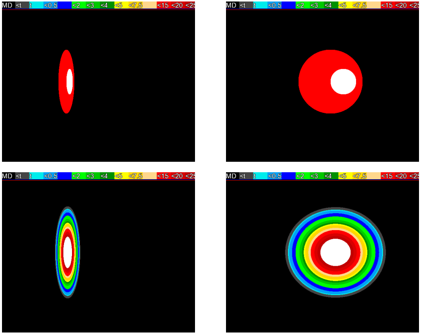
The GMM is defined so as to sum to 1, and the iterative method optimizes the likelihood of the parameters given the *positions* of the pixels (and not the intensity).
Two minor changes:

1. The total intensity associated with all the pixels in the image is used to scale the GMM
2. More intensive locations are repeated several (m) times:

$$m = 1 + \gamma round(\frac{CDF(I_{xy})}{freq(I_{mode})}) \forall I_{xy} < I_{mode} \tag{4}$$

Example Verification: Geometric

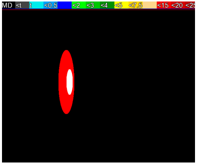
Geometric dataset from [Ahijevych et al., 2009]. Chose 3 Gaussians without much intensity correction



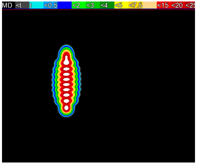
geom000 geom003

Geometric: why is the fit so bad?

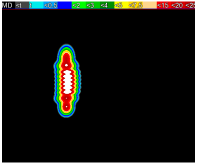
1. Abrupt changes in intensity: need more Gaussians
2. Fewer high-intensity pixels: need intensity correction



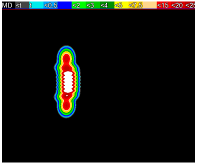
Orig



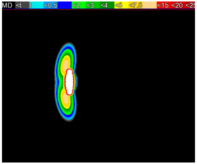
$\gamma = 0$



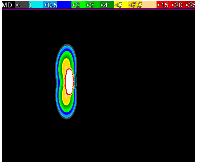
$\gamma = 0.5$



$\gamma = 1$



$\gamma = 3$



$\gamma = 5$

GMM fit better on real-world images

Example Verification: Geometric

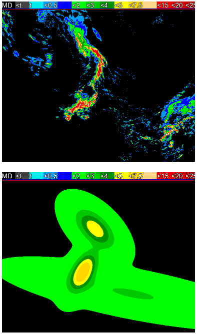
		μ_x	μ_y	σ_x^2	σ_{xy}	σ_y^2	π_k
0	Original	249	203	1720	4	128	49734
		249	203	1667	4	127	49734
		250	203	1668	9	127	49737
1	50 pts. right	249	**253**	1694	0	129	49731
		250	254	1682	4	121	49741
		250	253	1679	4	131	49732
2	200 pts. right	249	**404**	1612	4	126	49739
		250	403	1682	4	127	49735
		250	403	1760	0	129	49731
3	125 pts. right too big	250	**339**	1696	9	**2110**	**167034**
		249	340	1696	13	2048	167018
		250	341	1647	4	2021	167032

Example Verification: Geometric

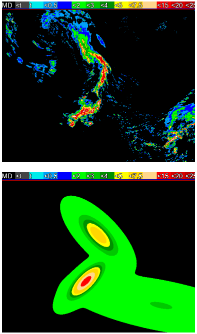
		μ_x	μ_y	σ_x^2	σ_{xy}	σ_y^2	π_k
4	125 pts.	249	**341**	**104**	1	**2046**	49736
	right	249	340	101	1	2027	49729
	turned	250	339	105	2	2120	49740
5	125 pts	249	**355**	1678	17	**8271**	**323126**
	right	250	356	1688	34	8203	323125
	huge	250	356	1668	16	8265	323121

Perturbed dataset

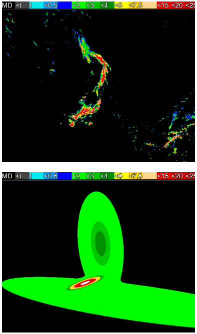
2km WRF from CAPS perturbed (See [Ahijevych et al., 2009]).



fake000



fake003



fake007

GMM parameters for perturbed cases

	μ_x	μ_y	σ_x^2	σ_{xy}	σ_y^2	π_k
Original	176	289	1305	743	1328	26437
	309	252	1272	482	665	26437
	379	407	1456	3919	20490	26437
3 pts. right	**181**	**292**	1306	743	1328	26437
5 pts. down	314	255	1270	490	675	26437
	384	410	1456	3918	20424	26437
6 pts. right	**186**	**295**	1307	744	1329	26437
10 pts. down	319	258	1269	496	675	26437
	389	414	1472	3928	20348	26437
12 pts. right	**195**	**299**	1206	840	1133	27101
20 pts. down	340	261	774	578	767	34201
	416	495	1051	1900	10252	17843

GMM parameters for perturbed cases

Description	μ_x	μ_y	σ_x^2	σ_{xy}	σ_y^2	π_k
24 pts. right	**212**	**311**	1059	813	1111	26527
40 pts. down	354	276	1239	802	837	33773
	432	483	1347	3110	13743	17566
48 pts. right	**250**	**335**	968	801	1121	25113
80 pts. down	387	304	1772	1052	934	33256
	452	447	1405	4659	20003	15666
12 pts. right	**192**	**298**	1096	859	**1198**	**33338**
20 pts. down	335	263	1178	773	829	42294
times 1.5	412	483	1264	2538	12634	22304
12pts. right	**222**	**306**	**2355**	194	**459**	**17815**
20 pts. down	345	258	79	162	486	20620
minus 2 mm	409	431	755	2884	20770	15932

Error measures

Translation error:

$$e_{tr} = \sqrt{(\mu_{xf} - \mu_{xo})^2 + (\mu_{yf} - \mu_{yo})^2} \tag{5}$$

Rotation error:

$$e_{rot} = \frac{180}{\pi} cos^{-1}(v_f . v_o) \tag{6}$$

v_f and v_o are the maximum-variance eigen vectors of the covariance matrices (Σ)

Scaling error:

$$e_{sc} = \frac{\pi_{k_f}}{\pi_{k_o}} \tag{7}$$

Associating Gaussians

For each Gaussian in the forecast field, compute the overall error with each Gaussian of observed field and pick the one with the lowest error.

$$\begin{aligned} e \; &= 0.3 * min(\tfrac{e_{tr}}{100}, 1) + \\ &0.2 * min(e_{rot}, 180 - e_{rot})/90 + \\ &0.5 * (max(e_{sc}, 1/e_{sc}) - 1) \end{aligned} \tag{8}$$

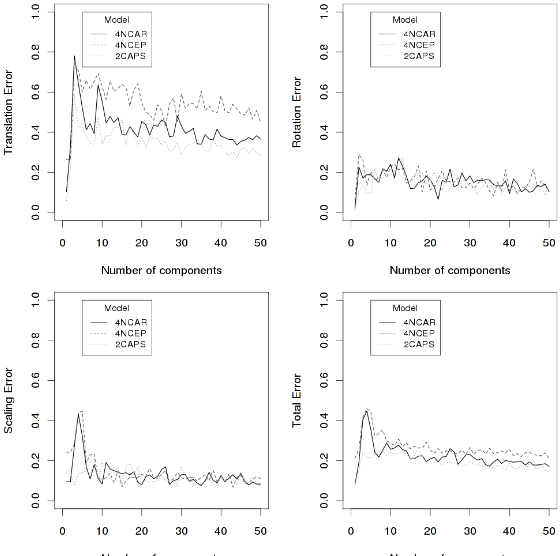
This can also be used to rank model forecasts.
The weights are subjective.

Synthetic images: Rank

	Description	e_{tr}	e_{rot}	e_{sc}	e
1	50 pts. right	**50**	0	1	0.15
2	200 pts. right	**201**	0	1	0.3
3	125 pts. right, too big	136	91	**3.36**	1.68
4	125 pts. right, wrong orient.	138	**90**	1	0.5
5	125 pts. right, huge	152	90	6.5	3.25

Rotation error on circular objects is undefined.
Ranking: geom001, geom002, geom004, geom003 and finally geom005.

June 1, 2005 Model runs



Acknowledgements

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The GMM fitting technique described in this paper has been implemented within the Warning Decision Support System Integrated Information (WDSSII; [Lakshmanan et al., 2007]) as part of the w2smooth process. It is available for download at www.wdssii.org.

References

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