A Bulk Microphysics Parameterization with Multiple Ice Precipitation Categories

JERRY M. STRAKA
School of Meteorology, The University of Oklahoma, Norman, Oklahoma

EDWARD R. MANSELL
Cooperative Institute for Mesoscale Meteorological Studies, The University of Oklahoma, Norman, Oklahoma

(Manuscript received 9 September 2003, in final form 27 September 2004)

ABSTRACT

A single-moment bulk microphysics scheme with multiple ice precipitation categories is described. It has 2 liquid hydrometeor categories (cloud droplets and rain) and 10 ice categories that are characterized by habit, size, and density—two ice crystal habits (column and plate), rimed cloud ice, snow (ice crystal aggregates), three categories of graupel with different densities and intercepts, frozen drops, small hail, and large hail. The concept of riming history is implemented for conversions among the graupel and frozen drops categories. The multiple precipitation ice categories allow a range of particle densities and fall velocities for simulating a variety of convective storms with minimal parameter tuning. The scheme is applied to two cases—an idealized continental multicell storm that demonstrates the ice precipitation process, and a small Florida maritime storm in which the warm rain process is important.

1. Introduction

Numerical modeling of storms is a mature field that has a history as long as the computers that have been capable of handling the computations. As computers have grown in power, researchers have added more complex and comprehensive physics into models that are being applied at ever-greater spatial resolution. A balance must still be struck between greater resolution and greater detail in the microphysical treatment—both of which result in increased computation. Microphysics schemes are generally categorized into “bulk” and “bin” approaches. The bin approach divides the particle spectrum into 20 or more size or mass bins (e.g., Berry 1967). A main disadvantage of bin microphysics is the memory and storage requirements for large storms in three dimensions, which becomes even greater if an electric charge is added for electrification studies. Bulk schemes, on the other hand, specify a functional form for the particle distribution and usually predict one or two characteristics of a particle category such as the total mass (mixing ratio) and concentration.

Over the past three or more decades cloud modelers have developed numerous types of bulk cloud and precipitation schemes. The simplest are saturation-adjustment schemes. The inadequacy of these very simple schemes for studying phenomena such as the development of cloud or rain led to parameterizations developed by Kessler (1969) and Simpson and Wiggert (1969). These predicted vapor, cloud droplets, and rain. Further expansion to study ice physics encouraged Wisner et al. (1972), Ogura and Takahashi (1973), Cotton (1972a), Cotton et al. (1982), Lin et al. (1983), and Rutledge and Hobbs (1984), to name a few, to develop schemes with two or three ice categories to capture some of the important physics associated with ice in precipitation production. These were expanded through the 1990s to take models to a greater understanding of liquid and ice physics with elaborate cloud and precipitation physics schemes and multiple moments. Ferrier (1994) presented a six-class two-moment scheme that included separate graupel and hail/frozen drops categories (and liquid water fraction on wet ice), improved treatments of some interactions, and other enhancements, such as a formulation that used generalized gamma distributions for describing particle spectra. The seven-class model of Walko et al. (1995) and Meyers et al. (1997) also had separate graupel and hail categories and additionally had separate snow and aggregates categories.

Inspired by the detailed cloud and precipitation path diagram of Braham (1968), a five-class scheme (Gilmore et al. 2004a, hereinafter GSR04) has been greatly expanded to include more hydrometeor habits...
than any previous bulk parameterization models to our knowledge. This responds to the need for a more general parameterization to simulate convective clouds over a range of latitudes (Tropics to high plains) as well as other precipitation systems that are not well simulated by a typical five-class scheme. For example, Ferrrier et al. (1995) showed the advantages of separate graupel and hail categories for producing more realistic storm characteristics. The scheme presented here has an emphasis on multiple ice categories in order to provide a smoother transition in physical characteristics, especially particle density and terminal velocity, as particles freeze or rime. The habits included in the model are cloud droplets, rain, ice crystals (three habits), aggregates/snow, graupel (three densities), frozen drops, and hail (small and large). Although the present paper describes a single-moment model, prediction of number concentration (double-moment model) is under development with optional prediction of mean diameter (triple-moment model), following Clark (1974).

Another motivation for the new expanded scheme was for studies of thunderstorm electrification. A host of laboratory and field studies have identified rebounding collisions between ice particles as the primary mechanism of charge separation in storms (MacGorman and Rust 1998, and references therein). Because of this primary dependence on ice particle interactions, a detailed ice microphysical package is desirable. Other schemes may be adequate for small storms (e.g., Helsdon and Farley 1987; Helsdon et al. 2001), but become less realistic for larger, more complex storms or must be tuned for a particular case. A scheme that needs little or no parameter tuning between storm types allows for the electrification parameterizations to remain constant as well. For example, charge separation rates are sensitive to particle fall speeds (more specifically, to collision impact speeds). With multiple precipitation ice types, fall speed variations can arise naturally as particles are converted between types, rather than artificially by tuning the particle characteristics for a particular simulation.

It is hoped with this new scheme, designed for use in three-dimensional cloud-resolving models, that it might be reasonable to make comparisons with radar, including polarimetric radar (Straka et al. 2000). Also, studies of other closely related phenomena, such as electrification and lightning, might be advanced. Last, the model physics might help to make studies of severe convection take hold of a new view of what might be causing, for example, large hail, large quantities of hail, rear-flank downdrafts, microbursts, and tornadoes. In the following parts of the paper the model is described.

2. 10-ICE microphysics model

The new microphysics package is in many regards a substantial expansion and improvement of Straka’s three-class bulk ice (3-ICE) scheme in GSR04, which is based on Lin et al. (1983). The newer 10-class bulk ice (10-ICE) scheme also uses a bulk representation for each hydrometeor type. It has the same two liquid hydrometeor categories (cloud droplets and rain) as the 3-ICE scheme and 10 ice categories that are characterized by habit and size—two pristine ice crystal habits [column ice (CI) and plate ice (IP), which have different minimum diameters for riming], rimed cloud ice (IR), snow (SA; here considered to be ice crystal aggregates), three categories of graupel with different characteristics [low-density graupel (GL), medium-density graupel (GM), and high-density graupel (GH)], frozen drops (F), small hail (H), and large hail (HL; Table 1. Thus, there are six large ice precipitation types (three graupel, frozen drops, and two hail) instead of the single graupel/hail category provided in the 3-ICE scheme, which includes only cloud ice, aggregates, and hail. The extra ice hydrometeor types were added to better represent the range of precipitation ice characteristics in a convective storm system. It was also developed to improve the treatment of conversions from one ice species to another with changes in habit, density, and terminal velocity.

The basic set of processes presented by GSR04 applies to the expanded scheme with a few exceptions. For example, the equation for accretion of cloud water by hail in GSR04 takes the same form for the graupel, frozen drops, and hail categories. Cloud droplets and cloud ice crystals (columns and plates) are treated as monodisperse (MD) distributions, and precipitation particles are assumed to have inverse exponential (IE) size distributions as in GSR04,

\[ n_s(D) = n_o \cdot e^{-D/\alpha_s}, \]

where \( n_s(D) \) (m\(^{-3}\)) is the number concentration of particles (per meter) with diameter \( D \) of hydrometeor category \( s \), and \( n_o \) is the fixed intercept value. Note that \( \alpha_s \) here is the characteristic diameter (e.g., as in Cotton et al. 1986), which is the inverse of the slope \( \lambda \) of the IE distribution, so that \( \alpha_s = \lambda^{-1} \). Table 1 lists the number concentration intercepts and the particle density for each hydrometeor category. Mass-weighted terminal fall speeds are plotted in Fig. 1 for an air density of 1.0 kg m\(^{-3}\).

A number of microphysical schemes now use gamma functions to describe the size distributions of one or more hydrometeor type (e.g., Ziegler 1985; Ferrier 1994; Waldo et al. 1995; Meyers et al. 1997), and log-normal functions have also been used (Clark 1976; Feingold and Levin 1986; Feingold et al. 1998). The gamma function has greater flexibility, especially in treating the small-diameter end of the size spectrum. Smith (2003) pointed out that observations are poor for small particles, making it difficult to distinguish observationally between the different predictions of gamma and exponential distributions. He further questioned...
whether the extra complication of a gamma function distribution is justifiable from a verification point of view. While the discussion by Smith (2003) somewhat justifies the use of exponential distributions for precipitation hydrometeors in the present version of the 10-ICE scheme, gamma functions would be more theoretically consistent for certain hydrometeor types. For example, the small- and large-hail categories each assume a minimum particle size (5 and 20 mm, respectively). Such changes are planned for future versions of the 10-ICE scheme. Other planned improvements are to use general gamma functions distributions for cloud microphysics and to predict number concentration of all ice species.

The hydrometeor mixing ratio conservation equation is (in vector form)

\[
\frac{\partial q}{\partial t} = -\frac{1}{\rho_a} \left[ \nabla \cdot (\mathbf{v}_a q \nabla) - q \nabla \cdot (\mathbf{v}_a \nabla q) \right] + \nabla \cdot (K_a \nabla q) \\
+ \frac{1}{\rho_a} \frac{\partial (\mathbf{V} \rho_a q)}{\partial z} + S. 
\]  

The first two terms on the right-hand side (in brackets) represent advection (the model scheme uses the flux formulation), followed by the turbulent mixing and fall-out terms. The microphysical source and sink terms \((S)\) include the following form and phase changes: condensation and evaporation, deposition and sublimation, freezing and melting, autoconversion of cloud to rain, ice aggregation, ice nucleation, and collection growth. To compute the source and sink terms, the model employs parameterized expressions for cloud microphysical processes proceeding from approaches of Lin et al. (1983), Cotton et al. (1986), Meyers et al. (1992), Ferrier (1994), and others, as described below.

Many processes are treated in 10-ICE in the same manner as in GSR04. These processes include, but are not limited to, stochastic freezing of raindrops (similar to Wisner et al. 1972, which is based on Bigg 1953), wet growth (shedding) of hail (and high-density graupel and frozen drops, but not snow and low- and medium-density graupel), vapor deposition and sublimation, saturation adjustment, melting, and evaporation.

\[\text{a. Collection rate equations}\]

The collection rate \((q_{ac})\) of one hydrometeor species \((y)\) by another \((x)\) follows GSR04. [The nomenclature has the interpretation \(q_{ac} = \text{rate of change of mixing ratio} \ q \text{ of species} \ x \text{ when it collects or accretes (ac) species} \ y\).] In the following expressions, \(d_a\) is the characteristic diameter (inverse slope) of an IE Marshall–Palmer size distribution, and \(D\) is the diameter for

<table>
<thead>
<tr>
<th>Category</th>
<th>Abbreviation</th>
<th>Intercept (n_{ac} \text{ (m}^{-3})</th>
<th>Density ((\text{kg m}^{-3})</th>
<th>(\mathbf{V}) or (C_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud droplets</td>
<td>W</td>
<td>—</td>
<td>1000</td>
<td>\</td>
</tr>
<tr>
<td>Column ice</td>
<td>CI</td>
<td>—</td>
<td>900</td>
<td>131.6 (D_{10}^{0.824} \sqrt{\frac{\rho_v}{\rho_a}})</td>
</tr>
<tr>
<td>Plate ice</td>
<td>IP</td>
<td>—</td>
<td>900</td>
<td>49.420 (D_{10}^{0.4150} \sqrt{\frac{\rho_v}{\rho_a}})</td>
</tr>
<tr>
<td>Rimed ice</td>
<td>IR</td>
<td>(1.0 \times 10^6)</td>
<td>300</td>
<td>\</td>
</tr>
<tr>
<td>Rain</td>
<td>R</td>
<td>(8.0 \times 10^6)</td>
<td>1000</td>
<td>\</td>
</tr>
<tr>
<td>Snow aggregate</td>
<td>SA</td>
<td>(8.0 \times 10^6)</td>
<td>100</td>
<td>\</td>
</tr>
<tr>
<td>Graupel (low)</td>
<td>GL</td>
<td>(4.0 \times 10^4)</td>
<td>300</td>
<td>0.8</td>
</tr>
<tr>
<td>Graupel (medium)</td>
<td>GM</td>
<td>(4.0 \times 10^5)</td>
<td>500</td>
<td>0.8</td>
</tr>
<tr>
<td>Graupel (high)</td>
<td>GH</td>
<td>(4.0 \times 10^5)</td>
<td>700</td>
<td>0.6</td>
</tr>
<tr>
<td>Frozen drops</td>
<td>F</td>
<td>(4.0 \times 10^4)</td>
<td>800</td>
<td>0.45</td>
</tr>
<tr>
<td>Small hail</td>
<td>H</td>
<td>(4.0 \times 10^4)</td>
<td>800</td>
<td>0.45</td>
</tr>
<tr>
<td>Large hail</td>
<td>HL</td>
<td>(1.0 \times 10^4)</td>
<td>900</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The Stokes law for cloud droplet fall velocity is valid for the allowed size range of \(5–50 \times 10^{-6}\) m. The kinematic viscosity \(\nu\) is given in appendix B. Ice plate and column fall velocities are from Davis and Auer (1974). Other fall velocity constants are \(a_r = 0.942, b_r = 0.8\) (Liu and Orville 1969) and \(\xi = 12.4\) and \(d_c = 0.42\) (Potter 1991). The base-state air density is denoted by \(\rho_v\), and the reference air density is \(\rho_a = 1.225 \text{ kg m}^{-3}\).
an MD distribution. Although number concentration \( n_x \) is not predicted, it is diagnosed \(( n_x = n_{ox} \cdot D_{ox})\) and used to calculate collision rates. Relevant equations explicitly include \( n_x \) for an easier future upgrade to two predicted moments. The collision rates also can be used for electrical charge transfer rates. For convenience, the accretion rate equations from GSR04 are repeated here. A future improvement will be the replacement of the collection equations with a more accurate method, such as that of Verlinde et al. (1990) or Ferrier (1994).

For an IE distribution \(( y_m \) collecting an MD distribution \(( y_e \)), the mixing ratio collection rate \( q_{x,ac} \) is

\[
q_{x,ac} = \frac{\pi}{4} E_{xy} q_{n_x} |\mathcal{V}_{x} - \mathcal{V}_{m} | \times \left[ \Gamma(3) D_{n,x}^2 + 2 \Gamma(2) D_{n,x} D_{y} + \Gamma(1) D_{y}^2 \right],
\]

where \( \Gamma \) is the complete Gamma function, \( \mathcal{V} \) is the (mass weighted) mean fall speed of an IE distribution, and \( \mathcal{V}_{m} \) is the fall speed of the MD particles.

For an IE distribution \(( y_e \) accreting another IE distribution \(( y_e \)), the mixing ratio collection rate is

\[
q_{x,ac} = \frac{\pi}{4} E_{xy} q_{n_x} |\mathcal{V}_{x} - \mathcal{V}_{y} | \times \frac{1}{\Gamma(4)} \left[ \Gamma(4) \Gamma(3) D_{n,x}^2 + 2 \Gamma(5) D_{n,x} D_{n,y} + \Gamma(6) D_{n,y}^2 \right].
\]

For an MD distribution \(( x_m \) interacting with an IE distribution \(( y_e \)), the mixing ratio collection rate is

\[
q_{x,ac} = \frac{\pi}{4} E_{xy} q_{n_x} |\mathcal{V}_{m,x} - \mathcal{V}_{y} | \times \frac{1}{\Gamma(4)} \left[ \Gamma(4) D_{x}^2 + 2 \Gamma(5) D_{x} D_{y} + \Gamma(6) D_{y}^2 \right].
\]

The 3-ICE scheme in GSR04 does not have any contact interactions between hydrometeors that are both MD distributions. In the 10-ICE scheme the MD ice crystal habits are allowed to collect cloud water droplets (also MD) to create rimed ice. Ice crystal plates and columns can also collect each other to form aggregates. For an MD distribution \(( x_m \) accreting another MD distribution \(( y_m \)), the mixing ratio collection rate is

\[
q_{x,ac} = \frac{\pi}{4} E_{xy} q_{n_x} |\mathcal{V}_{m,x} - \mathcal{V}_{m,y} | (D_{x}^2 + 2 D_{x} D_{y} + D_{y}^2).
\]

b. Differences between the 10-ICE and 3-ICE schemes

1) Cloud ice crystals

The cloud ice crystal (CI and IP) concentrations are diagnosed as in GSR04, except that cloud ice number concentration is determined from the Meyers et al. (1992) formulation for the number of active cloud nuclei rather than from Fletcher (1962). This practice allows the size of the crystals to vary and, thus, have a variable fall velocity. A drawback of this method is that crystal concentration may be seriously underdiagnosed at relatively high temperatures \(( -15^\circ \text{C} \) to \( 0^\circ \text{C} \)). Thus, there is a user input variable cimn that sets the minimum crystal concentration, subject to the minimum crystal mass \(( m_{l,\text{min}} = 6.88 \times 10^{-13} \text{ kg} \)). The value of cimn is usually set in the range of 1–200 \( \text{L}^{-1} \) \(( 10 \text{ L}^{-1} \) for the example simulations). The microphysical results do not appear to be particularly sensitive to the setting, because the main effect is at higher temperatures \(( -10^\circ \text{C} \) to \( 0^\circ \text{C} \)) where the ice crystal mixing ratios tend to be small. Nevertheless, collision rates with graupel are strongly influenced, and, thus, charge separation rates could be affected dramatically when electrification is activated in the model. A future improvement to the 10-ICE scheme will be the prediction of ice crystal and cloud droplet concentrations (e.g., following Ziegler 1985, for droplet concentration).

The habit (plate or column) of any new crystals is determined by the ambient temperature \( T_c \) (in degrees Celsius)—plates for \((-22.5 \leq T_c < -9 \) and \(-4 \leq T_c < 0 \)), and columns for \((-9 < T_c < -22.5 \) and \(-9 < T_c < -4 \)). These habit classifications will need to be revised in light of new work by Bailey and Hallett (2002), showing that polycrystals are a dominant habit at \( T_c < -20^\circ \text{C} \). The height–length relationship of ice crystal plates is \( H = 1.250L^{0.474} \) and for columns is \( D = 0.4764L^{0.958} \)
(Pruppacher and Klett 1978, converted to meters). The mass–diameter relationships for plates is \( D = 1.188M^{0.404} \) and for columns is \( L = 0.1871M^{0.5429} \) (Davis and Auer 1974; also converted to meters and kilograms). See also Straka et al. (2000) for a compilation of various height–length and length–mass relationships.

2) TERMINAL FALL VELOCITY

The mass-weighted mean terminal fall velocity \( \nabla \) of precipitating ice particles (graupel and hail) follows the form

\[
\nabla_x = \frac{\Gamma(4.5)}{6.0} \left( \frac{4g\rho_sD_{MAX}}{3C_{D,x}\rho_a} \right)^{1/2},
\]

where \( g \) is the acceleration due to gravity near the surface of the earth (9.8 m s\(^{-2}\)). The values of particle density \( \rho_s \) and drag coefficient \( C_{D,x} \) for graupel, hail, and frozen drops are also given in Table 1. Terminal fall velocities of the other particle categories are given in Table 1.

3) COLLECTION EFFICIENCIES

The efficiencies of an ice habit collecting another ice habit are generally smaller in the 10-ICE scheme than for 3-ICE. Snow-accreting cloud ice (crystals or rimed ice) follows Ferrier (1994) with \( E_{\text{gr},x} = 0.01 \exp(0.1T) \), where \( T \) is the temperature in degrees Celsius. For dry graupel or hail accreting cloud ice or snow aggregates, \( E_{\text{gr},x} = E_{\text{gr},x} = 0.01 \exp(0.1T) \) (Ferrier et al. 1995). For graupel or hail particles in wet growth mode, \( E_{\text{gr},x} = E_{\text{gr},x} = 1.0 \). The efficiency of rain-collecting cloud ice \( E_{\text{ra},x} \) is unity for cloud ice diameters greater than 40 \( \mu \)m and zero for smaller diameters.

The efficiency for cloud droplet collection by precipitation particles (rain, graupel, hail, and frozen drops, excluding snow aggregates) is given by a least squares fit of the experimental data published by Mason (1971, his Table A.2),

\[
E_{\text{sw}} = \text{Min}[-0.27544 + 0.26294 \times 10^6R_w - 1.8896 
\times 10^{10}R_w^2 + 4.4626 \times 10^{14}R_w^3, 1.0],
\]

where \( R_w \) is the droplet radius (in meters) and the collector has an assumed radius of 1000 \( \mu \)m. The minimum droplet radius is 5 \( \mu \)m, corresponding to \( E_{\text{sw}} = 0.62 \). The efficiency for cloud droplet collection by small ice particles and snow (i.e., CI, IP, IR, and SA as listed in Table 1) is \( E_{\text{sw}} = 0.5 \) if both the cloud droplet has radius \( R_w > 7.5 \) \( \mu \)m and the ice particle diameter exceeds a threshold value (\( D_{\text{ci}} > 30 \) \( \mu \)m, \( D_{\text{ip}} > 300 \) \( \mu \)m, \( D_{\text{ir}} > 30 \) \( \mu \)m, and \( D_{\text{sa}} > 100 \) \( \mu \)m); otherwise \( E_{\text{sw}} = 0.0 \).

4) ICE DEPOSITION (SUBLIMATION)

Unlike GSR04 or Lin et al. (1983), deposition and sublimation are calculated explicitly for ice crystals as well as all other ice categories (e.g., as in Ferrier 1994).

Therefore, the Bergeon process is not treated as a separate process, but it is represented implicitly by the balance between the individual condensation and deposition growth terms.

The general form for deposition or sublimation (ds) of vapor (v) to ice hydrometeor category \( x \) is

\[
q_{\text{xsv}} = \frac{4\pi}{\rho_a} (S_x - 1) - \frac{n_x V\text{ENT}_x C_x}{K_{\text{s},R_a} T^2} + \frac{1}{\rho_a q_{\text{vp},x}},
\]

\[
q_{\text{dpv}} = \text{Max}(q_{\text{xsv}}, 0), \quad \text{and}
\]

\[
q_{\text{sbv}} = \text{Min}(q_{\text{xsv}}, 0),
\]

where \( q_{\text{xsv}} \) is the saturation vapor mixing ratio with respect to ice, and \( S_x = q_x/q_{\text{vp}}, \) is the ratio of vapor mixing ratio to the ice saturation mixing ratio. Capacitance \( C_x \) and \( V\text{ENT}_x \) are given in appendix B along with the definitions of the other variables. Equation (9) represents deposition (\( q_{\text{dpv}} \)) when it is positive (i.e., \( S_x - 1 > 0 \)) and sublimation (\( q_{\text{sbv}} \)) for negative values. The formulation is identical to that in GSR04 for hail and snow aggregates.

5) SATURATION ADJUSTMENT

The saturation-adjustment scheme of Tao et al. (1989) is employed to approximate the nonlinear interactions between bulk water substance phase changes and the atmospheric heat and water vapor content. The scheme differs from Tao et al. (1989) mainly in the temperature dependence terms that determine the fraction of vapor adjustment that acts on droplets and ice crystals (CND and DEP, respectively, in Tao et al. 1989, originally from Lord et al. 1984). Because the interaction between vapor and ice crystals is explicitly treated (above), the adjustment scheme acts only on cloud droplets for \(-20^\circ < T < 0^\circ \)C by redefining CND and DEP as

\[
\text{CND} = \text{Max} \left( \text{Min} \left( 1, \frac{T - T_{\text{eo}}}{20} \right), 0.0 \right) \quad \text{and}
\]

\[
\text{DEP} = 1 - \text{CND},
\]

where \( T_{\text{eo}} = 233.15 \) K. When the scheme causes vapor deposition to (or sublimation from) cloud ice, it is distributed between the plate and column habits according to their fractions of the total ice crystal mass at a given point. Note that cloud ice initiation is still allowed for \( T > -20^\circ \)C via primary nucleation or ice multiplication. Tests performed with the CND formulation from Tao et al. (1989), however, show only minor differences from (12).

c. Precipitation-sized ice conversions

1) MULTICOMPONENT CONVERSIONS DUE TO RIMING

Ice precipitation conversions resulting from accretion of cloud droplets (i.e., riming) is a central feature of the
10-ICE model. Mass can be shifted among the frozen drops and three graupel categories and from snow to graupel depending on the Lagrangian growth history of the collector particles. In Ferrier (1994) it was assumed that riming ages of snow, graupel, and frozen drops and the amount of mass collected were some of the key parameters in the multicomponent conversions. That scheme was unique in its design and can be considered the first to deviate from the standard three-component collection interaction.

In Ferrier (1994), particles were assumed to have been riming at the current rate during the previous 120 s. A problem with this assumption is that particles may be converted too quickly to the next category when they first appear or first enter a higher- or lower-density riming region. For example, consider a particle with a 5 m s⁻¹ fall speed being carried upward in a strong updraft of 30 m s⁻¹, so that it may encounter quite a range of riming density regimes and would rise some 3000 m in 120 s. Sudden large riming rates might lead to significant changes where realistically the density changes take some time and the particle may move some 1000 m or more before a transfer should occur. As proposed by Straka and Rasmussen (1997), the approach of Ferrier (1994) is refined here by replacing the fixed riming time with a variable riming age. Explicitly calculating a riming age helps to make more accurate conversions and avoid converting particles too rapidly.

The goal of the new scheme is to maintain the average particle density of a category by transferring particles to higher- or lower-density categories if the mean riming rates and riming ages indicate a sufficient shift in average density. Each category can both receive and lose particles in the same step (except snow, which can only retain or lose particles). For example, the collection of supercooled cloud water by high-density graupel may result in increased medium-density graupel content, while simultaneously frozen drops are converted to high-density graupel. The model uses an extension of the method of Straka and Rasmussen (1997) to determine how long a particle category has been actively riming (i.e., the riming history). We start with the equation for a Lagrangian variable \( \tau_a \) for the “age” of an air parcel (Straka and Rasmussen 1997):

\[
\frac{d\tau_a}{dt} = S_{w,a},
\]  

(14)
or, in Eulerian form,

\[
\frac{\partial \tau_a}{\partial t} = -\mathbf{V} \cdot \nabla \tau_a + S_{w,a},
\]  

(15)

where \( S_{w,a} \) is a source term that determines whether the parcel age is increasing (\( S_{w,a} = 1 \)) or not (\( S_{w,a} = 0 \)), and \( \mathbf{V} \) is the wind vector. In Straka and Rasmussen (1997), \( \tau_a \) is used to determine how long a parcel has had a cloud droplet mixing ratio \( q_c \), greater than a threshold (e.g., \( 1 \times 10^{-5} \text{ kg kg}^{-1} \)). In that case, \( \tau_a = 0 \) and \( S_{w,a} = 0 \) until the minimum \( q_c \) appears, after which \( S_{w,a} = 1 \) and \( \tau_a \) begins to increase. When \( q_c \) drops below threshold, \( S_{w,a} = 0 \) and \( \tau_a \) may be set to zero. Also, in the Eulerian form, \( \tau_a \) can be thought of as telling how long the air parcel at a particular grid point has carried a minimum droplet content. This method is used to set a minimum existence time for the droplets before autoconversion to rain is allowed ([48]).

Now, the same method can be applied to a time-based particle property such as the length of time \( \tau_s \) that the particles have been actively riming. The equation for riming age \( \tau \) of ice hydrometeor category \( x \) (graupel, frozen drops, and aggregates) simply adds a term to (15) for the sedimentation of particles relative to their air parcel:

\[
\frac{\partial \tau_x}{\partial t} = -\mathbf{V} \cdot \nabla \tau_x + \frac{\partial (\nabla \cdot \tau_x)}{\partial z} + S_{w,x},
\]  

(16)

where \( \nabla \cdot \tau_x \) is the mass-weighted mean terminal velocity. The source–sink term \( S_{w,x} \) is again equal to unity for riming rates above the minimum threshold and zero for riming rates below threshold (typically \( 1.0 \times 10^{-12} \text{ s}^{-1} \)). When the riming rate falls below the threshold, \( \tau_s \) can be set to zero.

A five-dimensional lookup table is used to determine the fraction \( f_{w,y} \) of particles (and their collected rime) in category \( x \) that are converted to category \( y \) as a result of collecting cloud water droplets \( w \). The conversions of newly accreted rime and preexisting particle mass are determined separately. The lookup table is a function of collector (input) category mixing ratio \( q_x \), cloud droplet mixing ratio \( q_w \), temperature \( T \), output category \( y \), and riming age \( \tau_x \) of the input category:

\[
q_{xw}\mathbf{a} = f_{w,y}(q_x, q_w, T, \tau_x) \times q_{xw} \]  

and

\[
q_{x}\mathbf{a} = f_{w,y}(q_x, q_w, T, \tau_x) \times q_{xw},
\]  

(17)

(18)

where \( y \) is an integer value that indicates the output category. The first quantity \( q_{xw}\mathbf{a} \) ([17]) is the fraction of rime collected by \( x \) (\( q_{xw} \)) and is converted to category \( y \). (Note that the quantity \( q_{xw} \) is the fraction of \( q_{xw} \) that remains as a source for species \( x \).) The second quantity \( q_{x}\mathbf{a} \) ([18]) is the part of the preexisting mixing ratio of \( x \) that is converted to \( y \) through riming (accretion) of cloud water. The reverse collection rate \( q_{w}\mathbf{a} \) determines the part of the starting value of \( q_x \) that can be converted to another category. For appreciable riming rates, \( q_{w}\mathbf{a} \) is generally equal to the maximum value of 0.1 \( q_x \) (i.e., maximum depletion rate of 10% per time step). Practically speaking, \( q_{w}\mathbf{a} \) acts as a kind of Heaviside function that turns on when conversion of preexisting particles if they are experiencing appreciable riming. Depending on conditions, values of \( f_{w,y} \) may retain a large fraction of accreted rime in the collector category if there is not a sufficient change in particle density (i.e., \( f_{w,y} \) may be close to unity). A separate lookup table is created for each input category (SA, GL, GM, GH, and F). Riming of SA can produce
mass as SA, GL, GM, GH, and F, but categories GL, GM, GH, and F are not converted to SA by riming, following the same reasoning as Ferrier (1994) that riming would not be able to reduce particle density below that of low-density graupel. Hail particles (H and HL) are assumed to maintain their mean density and are not converted to lower densities by riming. Although it is possible for one particle type to have conversions to more than one other type (e.g., medium-density graupel converted to both high-density graupel and frozen drops), the riming conversions are usually either to the next higher- or lower-density category only.

The construction of the ice conversion lookup table is similar in spirit to that of Feingold et al. (1998) in using a bin microphysics approach. The collector particle spectrum is split into *k* logarithmically spaced size bins (e.g., Berry 1967; Farley 1987):

\[
m_x(i) = m_{0,x} + \frac{3(i-1)}{J_0}; i = 1, k,
\]

\[
D_x(i) = \left[ \frac{6m_x(i)}{\sigma_{px}} \right]^{1/3},
\]

\[
n_i(i) = n_{0,x} \exp \left[ - \frac{D_x(i)}{D_{px}} \right] \Delta(i),
\]

where \(D_{px}\) is the characteristic diameter corresponding to the table value of \(q_x\) and \(\Delta\) is the width of the diameter bin [\(\Delta(i) = D_x(i)/J_0\)]. Using values of \(m_{0,x} = 4.7 \times 10^{-10}\) kg, \(J_0 = 7.5\), and \(k = 41\) yields a mass range of \(4.7 \times 10^{-10}\) to \(4.17 \times 10^{-3}\) kg. The terminal fall velocity of the \(i\)th bin is found as

\[
V_x(i) = \left[ \frac{4g\rho_x D_x(i)}{3C_{D,x}\bar{\rho}_a} \right]^{1/2} \text{ (GL, GM, GH, FD)}
\]

and

\[
V_x(i) = c_x[D_x(i)]^{1/2} \left( \frac{\rho_x}{\bar{\rho}_a} \right) \text{ (SA)},
\]

where \(\rho_a = 1.225\) kg m\(^{-3}\) and \(\bar{\rho}_a\) is determined from the input reference pressure at \(T = 0\)°C. Note that these velocities are the integrands for the corresponding mass-weighted velocities of Table 1 and (7). Continuous collection of cloud water droplets is then assumed for each bin to determine the mass and mixing ratio gain rates:

\[
\frac{dm_x(i)}{dt} = \frac{\pi}{4} [D_x(i) + D_w]^2 E_{x,w} \bar{\rho}_a [V_x(i) - V_w]
\]

and

\[
\frac{dq_x(i)}{dt} = \frac{n_i(i) dm_x(i)}{\bar{\rho}_a}.
\]

The density of the rime \(\rho_{x,\text{riming}}(i)\) added to mass bin \(i\) of collector ice category \(x\) is calculated by an adjusted formula from Macklin (1962). It is assumed that the cloud droplet collection efficiency \(E_{x,w} = 1\). The coefficient and power values for the rime density equation are those found by Heymsfield and Pflaum (1985), and the relative fall velocity is adjusted from the stagnation point value \([V_x - V_w]\) by a factor of 0.6 to get the approximate average impact velocity (Rasmussen and Heymsfield 1985). The value of 0.6 arises because most collisions are glancing ones rather than centerline or head-on collisions:

\[
\rho_{x,\text{riming}}(i) = 300 \left( \frac{0.5 \times 10^6 D_w [0.6 [V_x(i) - V_w]]}{-(T - T_0)} \right)^{0.44},
\]

where \(T_0 = 273.15\) K and \(D_w\) is the cloud droplet diameter (converted to radius in micrometers by the factor \(0.5 \times 10^6\)). The new average particle density \(\bar{\rho}_{x,w}(i)\) for mass bin \(i\) is then

\[
\frac{dm_{x,w}(i)}{dt} = \frac{\pi}{4} [D_x(i) + D_{w,M}]^2 E_{x,w} \bar{\rho}_{x,w} [V_x(i) - V_w]
\]

\[
\frac{dq_{x,w}(i)}{dt} = \frac{n_i(i) dm_{x,w}(i)}{\bar{\rho}_{x,w}}.
\]

The “Max” and “Min” functions apply a lower bound of 100 kg m\(^{-3}\) and an upper bound of 900 kg m\(^{-3}\) to the rime density. If the new particle density for a mass bin changes sufficiently, then the bin as a whole is assigned to the output particle category associated with the new density. (If riming is insufficient to change significantly the particle density then the mass stays in the source category.) After the new particle density and output category have been calculated for each bin, the total mass fraction transferred to each output category is calculated. The calculation process is repeated for each table value of \(q_x\) and \(q_{w,M}\) (both have a range of 0–10 g kg\(^{-1}\), with increments of 0.5 g kg\(^{-1}\)), \(T\) (233–273 K, with increments of 2 K), and \(\tau_w\) (20–120 s, with increments of 20 s). All cloud droplets are assumed to freeze homogeneously at \(T < 233.15\) K, preventing riming at lower temperatures. The extrema table values are used if an input value (\(q_{x,M}, q_{w,M}\), or \(\tau_w\)) exceeds the range of tabulated values.

2) MULTICOMPONENT CONVERSIONS DUE TO RAIN ACCRETION

Precipitation ice particles may be converted to particles of different mass density when they collect supercooled raindrops. Although binary coalescence of similarly sized precipitation particles (e.g., rain–rain or rain–ice) is most rigorously treated by the stochastic coalescence equation (e.g., Ziegler 1985), we follow several previous modeling studies by employing the continuous collection approximation based on mean terminal speed difference of the collector and collected particles. The conversions are similar to the riming conversions, except that they are instantaneous (i.e., no consideration of rain collection time history, and \(\Delta t = \ldots\))
one time step). A three-dimensional lookup table is constructed in a similar manner as for riming conversions, but for only one time interval and one temperature (significant quantities of supercooled raindrops are assumed to exist only in the range from $-10^\circ$ to $0^\circ$C). In the rain collection case, both the ice ($x$) and rain ($r$) particle spectra are broken up into $k$ logarithmic-sized bins. Then the density change is found for each ice particle size collecting (or being collected) by each size of raindrop. The result is a two-dimensional array of mass gain rates. The mass and mixing ratio gain rates for ice category $x$ collecting rain are

$$\frac{dm_r}{dt}(i, j) = \frac{\pi}{4} [D_s(i) + D_l(j)] E_{sr} \rho_r q_r(j) \max(0, [V_s(i) - V_r(j)])$$

and

$$\frac{dq_{x,y}}{dt}(i, j) = \frac{n_s(i) dm_r}{\rho_s} \frac{dt}{(i, j)}$$

so that $dq_{x,y}/dt = 0$ if the rain particle has a larger fall velocity than the ice category. For rain-collecting ice category $x$ the rates are

$$\frac{dm_r}{dt}(i, j) = \frac{\pi}{4} [D_s(i) + D_l(j)] E_{sr} \rho_r q_r(i) \max(0, [V_s(i) - V_r(j)])$$

and

$$\frac{dq_{x,y}}{dt}(i, j) = \frac{n_s(i) dm_r}{\rho_s} \frac{dt}{(i, j)},$$

where the collection efficiency $E_{sr}$ is assumed to be unity. In this case $dq_{x,y}/dt = 0$ if $V_r(i) > V_r(j)$. The maximum mass-weighted density of a combined ice particle (bin $i$) and rain particle (bin $j$), assuming no freezing of the liquid, is

$$\rho_{new}(i, j) = \frac{m_s(i) \rho_s + m_r(j) \rho_r}{m_s(i) + m_r(j)}.$$

Each value of $\rho_{new}(i, j)$ is checked for reassignment to a different category with the appropriate particle density range. The fraction $f_{x,y}$ is the portion of rain mass collected by ice category $x$ that is transferred to ice category $y$ (where $x$ is one of SA, GL, GM, GH, or F, and $y$ is one of GL, GM, GH, or F). Conversely, $f_{x,y}$ denotes the fraction of qracy that is transferred to category $y$. (Note that qracy is the collection of $x$ by $r$.) Let $K_x$ be the set of points $(i, j)$ where $\rho_{new}(i, j)$ is in the density range of category $y$ [i.e., $\rho_{y,\min} < \rho_{new}(i, j) \leq \rho_{y,\max}$], then the conversion fractions are

$$f_{x,y} = \left[ \sum_{(i, j) \in K_x} \frac{dq_{x,y}}{dt}(i, j) \right] / \left[ \sum_{(i, j) \in K_x} \frac{dq_{x,y}}{dt}(i, j) \right]$$

and

$$f_{x,y} = \left[ \sum_{(i, j) \in K_x} \frac{dq_{x,y}}{dt}(i, j) \right] / \left[ \sum_{(i, j) \in K_x} \frac{dq_{x,y}}{dt}(i, j) \right],$$

where the denominators are the sums over all points $(i, j)$. Last, we have

$$q_{x,\text{new}} = f_{x,y} q_{x, r} \times q_{x, \text{rime}}$$

and

$$q_{x,\text{rime}} = f_{x,y} q_{x, r} \times q_{x, \text{rime}},$$

where $q_{x,\text{rime}}$ is the rate of rain ($r$) accreted by ice category $x$, producing ice category $y$ (i.e., conversion of $r$ to $y$), and $q_{x,\text{rime}}$ is the rate of rain accreting $x$, producing $y$ (actually the conversion of preexisting particles in ice category $x$ to ice category $y$).

3) CONVERSION OF LARGE-PRECIPITATION ICE

Particles of graupel and frozen drops (GL, GM, GH, and F) that exceed $D_{hl} = 5 \text{ mm}$ can be converted to H at the rate $q_{hl}x$, where $h$ is the small-hail category and $x$ is the source category. In addition, small-hail particles larger than $D_{hl} = 20 \text{ mm}$ are converted to large hail ($q_{hl}hn$). In general, then, the rate of mixing ratio $(q)$ conversion (cn) of the larger particles of type $x$ (GL, GM, GH, F, or H) to category $y$ (H or HL) is given by

$$q_{ycn} = \frac{\text{Mass of particles with } D > D_{hn}}{\rho_s \Delta t} = \frac{n_{x,a} \rho_s}{6 \rho_{a} \Delta t} \int_{D_{hn}}^{\infty} D^3 e^{-D/\rho_{a} \Delta t} dD$$

$$= \frac{n_{x,a} \rho_s}{6 \rho_{a} \Delta t} \varepsilon^{-D_{hn}/\rho_{a} \Delta t}(D_{hn}^3 + 3D_{hn}^2 \rho_{a} \Delta t)$$

$$+ 6D_{hn}^2 \rho_{h} \Delta t + 6D_{hn}^3 \rho_{h} \Delta t),$$

where $\Delta t$ is the model time step (to convert the mixing ratio to a rate), $D_{hn} = D_{hl}^x$ for conversion of GL, GM, GH, or F to H, and $D_{hn} = D_{hl}^x$ for converting H to HL. The conversion of frozen drops and small hail occurs without any other conditions, but graupel (GL, GM, or GH) is converted to hail only when the bulk average density due to accreted rime $\bar{\rho}_{x,\text{rime}}$ and the bulk particle density due rain accretion $\bar{\rho}_{x,r}$ are both greater than 800 kg m$^{-3}$. The time-weighted particle density $\bar{\rho}_{x,\text{rime}}$ due to riming is also calculated in the model as a bulk quantity for graupel particles,

$$\bar{\rho}_{x,\text{rime}} = \frac{q_x \rho_x + \min(\tau_{x,\text{rime}}, \rho_{x,\text{rime}})}{q_x + \min(\tau_{x,\text{rime}})} q_{x, \text{rime}} \bar{\rho}_{x,\text{rime}},$$

where

$$\bar{\rho}_{x,\text{rime}} = 300 \frac{0.5 \times 10^6 D_n(0.6[T - T_{c}])}{[T - T_{c}]} 0.44,$$

$D_n$ is cloud droplet diameter, and $\tau_{x}$ is a maximum riming age (usually set to the maximum table value of 120 s). The bulk particle density $\bar{\rho}_{x,\text{rime}}$ due to rain accretion is
The self-collection rate for ice crystal aggregation to snow is calculated following the Cotton et al. (1986) adaptation of the aggregation model 1 in Passarelli and Srivastava (1979):

\[ q_{\text{ac}i} = 0.25 \pi \frac{D_i}{6n_i} E_i D_i^2 V_i, \]

where \( n_i \) is the ice crystal mass, \( D_i \) is the crystal diameter (or, for rimed ice, the characteristic diameter \( D_{i,\text{w}} \)), and \( V_i \) is the terminal fall velocity. [Note that \( q_{\text{ac}i} = \) production of snow \( s \) by conversion \( \text{cn} \) of ice crystals \( i \).] The ice–ice collection efficiency is \( E_i = 0.1 \exp(0.1 T_c) \). The same equation is applied to ice crystal columns and plates and to rimed ice. The rates of one ice crystal type collecting another are treated also as source terms for snow aggregates. The cross collections are treated as MD–MD or MD–IE distribution collection equations, as appropriate.

2) INITIATION OF PRECIPITATION BY THE ICE PROCESS

The ice process of precipitation formation begins with the riming of pristine cloud ice crystals (IP and CI) to become small IR and GL. The accumulated mass of rime per ice crystal \( m_{\text{rime}} \) is calculated from the current crystal-riming rate \( q_{\text{ac}w} \) and the riming age \( \tau_i \):

\[ m_{\text{rime}} = \frac{\rho_i \tau_i}{n_i} q_{\text{ac}w}. \]

Mass transfers to categories IR and GL are calculated from the mass fraction of rime \( f_{\text{rime}} \):

\[ f_{\text{rime}} = \min\left(1, \frac{m_{\text{rime}}}{m_i}\right), \]

where \( m_i \) is the ice crystal (IP or CI) mass. Conversion is initiated when the following two conditions are met: 1) \( f_{\text{rime}} > 0.1 \), and 2) the riming rate \( q_{\text{ac}w} \) exceeds the greater value of \( 10^{-12} \text{ s}^{-1} \) and the vapor deposition rate \( q_{\text{dpu}} \) (10), similar to the approach of Reisner et al. (1998). The assumption is that the crystals will not change substantially in character until an appreciable fraction of their mass comes from riming and the riming mass gain rate is greater than the vapor deposition rate. The portion of newly acquired rime on the crystals that can be converted to IR and GL is \( q_{\text{ac}w}0 \),

\[ q_{\text{ac}w0} = \max(q_{\text{ac}w} - q_{\text{dpu}}, 0). \]

As in the graupel conversions, the portion of the pre-existing crystal mixing ratio that is available for conversion \( (q_{\text{cn}0}) \) is determined from the reverse collection rate \( q_{\text{waci}} \):

\[ q_{\text{cn}0} = \min\left(q_{\text{waci}} \times \frac{q_{\text{ac}w0}}{q_{\text{ac}w}}, 0.1 q_i\right). \]

The available rime \( q_{\text{ac}w0} \) and pre-existing crystal mixing ratio \( q_{\text{cn}0} \) are partitioned into rimed ice and low-density graupel. The fraction of each that is converted to GL is \( f_{\text{rime}} \), and the part going to IR is \( (1 - f_{\text{rime}}) \). The continuous shift between IR and GL provided by \( f_{\text{rime}} \) parameterizes the increasingly graupel-like characteristics of heavily rimed crystals. The same procedure is also applied to IR for conversion to GL, except that there is only one output category (GL) for the converted mass, and so there is no need for \( f_{\text{rime}} \).

3) ICE MULTIPLICATION AND ENHANCEMENT

The model also includes a Hallett–Mossop ice multiplication parameterization (Hallett and Mossop 1974). The scheme follows Cotton et al. (1986) (type I, type II) but employs the stair-step temperature function \( (H_{M_F}) \) of Ferrier (1994) instead of the triangle-shaped function \( f(T_c) \) in Cotton et al. (1986). Both the type-I (350 splinters formed for every \( 1 \times 10^{-3} \text{ g of cloud water accreted} \) and type-II (one splinter formed for every 250 cloud droplets larger than 24 \( \mu \text{m in diameter accreted by graupel or hail} \) schemes of Cotton et al. (1986) are included. We generally use only the type-II term because it seems to have more laboratory support (Mossop 1976, 1985). The type-II parameterization treats the droplet distribution as a gamma function of volume, with a mean diameter being the same as the monodisperse diameter (Cotton et al. 1986). (Droplets are otherwise treated as monodisperse, except for autoconversion.) The mass of the ice splinters has a default value of \( 5 \times 10^{-10} \text{ kg} \). The Hobbs–Rango enhancement parameterization of Ferrier (1994) is also included (Hobbs and Rango 1985; Hobbs 1990; Rango and Hobbs 1991).

4) CLOUD ICE INITIATION

Primary ice initiation follows Meyers et al. (1992) and Ferrier (1994). In addition, contact-freezing nucleation of cloud droplets is the same as in Meyers et al. (1992).
5) Cloud water to rain conversion

The autoconversion scheme uses the method of Straka and Rasmussen (1997) to determine the Lagrangian parcel age (15). This Lagrangian information is then used to delay the conversion of droplets to rain until a minimum mixing ratio of cloud water ($q_{w,\text{min}} = 0.01 \text{ g kg}^{-1}$) has existed in the parcel for a given time $\tau_a > \tau_{\text{a,o}}$ [typically $\tau_{\text{a,o}} = 5$ to 10 min, and $\tau_a$ is determined by (16)]. Other than the delay time, the autoconversion of droplets to rain follows Ferrier (1994), which has a modified version of Berry (1968) as adapted by Orville and Kopp (1977). Ferrier (1994) used a minimum droplet size $D_{w,o}$ to set the threshold mixing ratio $q_{w,o}$:

$$q_{w,o} = \frac{\pi}{6} n_w D_{w,o} \frac{\rho_w}{\rho_a},$$

$$q_{\text{diff}} = \text{Max}[0, (q_w - q_{w,o})], \quad \text{and} \quad (46)$$

$$q_{\text{rcnw}} = \frac{\delta_r (1 \times 10^{-3} \rho_w q_{\text{diff}})}{1.2 \times 10^{-4} + \frac{1.596 \times 10^{-17}}{\phi \rho_a q_{\text{diff}}}} ,$$

where $D_{w,o} = 20 \times 10^{-6} \text{ m}$, $\phi$ is the assumed droplet dispersion (typical value of 0.15–0.3), and

$$\delta_r = \begin{cases} 0 & \text{for } \tau_a < \tau_{\text{a,o}} \\ 1 & \text{for } \tau_a \geq \tau_{\text{a,o}} \end{cases} \quad (48)$$

The factors of $1 \times 10^{-3}$ are included for conversion from the SI variable units to the cgs equation units, preserving the original constants.

3. Example results

Two storm environments were used to illustrate the treatment of the two modes of precipitation formation commonly referred to as the warm and cold rain processes. In the warm rain process, raindrops develop by the collision and coalescence of cloud droplets. The raindrops may freeze if an updraft carries them above the melting level, and the resulting frozen drop may subsequently be converted by riming into other ice particle types. The cold rain, or ice precipitation, process begins with the appearance of ice crystals, either by the freezing of liquid cloud droplets or initiation by vapor deposition onto an ice nucleus. A few of the ice crystals grow large enough to start collecting cloud droplets and develop into graupel. Graupel particles eventually fall below the melting level to become rain. A continental multicell storm illustrates the ice process, and the warm rain process is demonstrated in a Florida (maritime) storm simulation. For evaluating model precipitation fields, the estimation of radar reflectivity follows Ferrier (1994), with the simplifying assumption that all ice particles are dry. For comparison, the multicell storm was also simulated with the 3-ICE microphysical scheme, and results are presented in appendix A.

a. Model numerics and dynamics

The numerical simulation model is nonhydrostatic and fully compressible (Straka 1989; Straka and Anderson 1999b) and is based on the set of equations described by Klemp and Wilhelmson (1978). Prognostic equations are included for three momentum components—pressure, potential temperature, and turbulent kinetic energy (see Carpenter et al. 1998). The advection and diffusion numerics in the model all include a conservation principle. For momentum advection, the model uses a time-centered, quadratic (i.e., energy) conserving "box" scheme (Kurihara and Holloway 1967) in the vertical direction, and a sixth-order local spectral scheme in the horizontal (Straka and Anderson 1993a). Scalar advection is performed with a forward-in-time, sixth-order, flux divergence-corrected Crowley scheme (Tremback et al. 1987) with a monotonic filter (Leonard 1991). The same sixth-order Crowley method is applied in the vertical direction to treat the hydrometeor fallout terms, resulting in much less computational smoothing than from a first-order treatment. The turbulent mixing parameterization is based on Klemp and Wilhelmson (1978), Deardorff (1980), and Moeng (1984) (see Carpenter et al. 1998).

b. Continental storm

A continental multicell storm simulation exhibits the ice precipitation process in the 10-ICE microphysics. An analytical thermodynamic sounding was used following Weisman and Klemp (1982), with a boundary layer vapor mixing ratio of 13.5 g kg$^{-1}$, resulting in convective available potential energy (CAPE) of about 1630 J kg$^{-1}$ and convective inhibition (CIN) of 44 J kg$^{-1}$. The environment had a half-circle hodograph ($U_z = 20 \text{ m s}^{-1}$) with the wind shear confined to the lowest 5 km, as in Weisman and Klemp (1984). The relative humidity profile $H(z)$ is slightly reduced from that of Weisman and Klemp (1982):

$$H(z) = \begin{cases} 1 - \alpha \left( \frac{z}{z_{\text{tr}}} \right)^{5/4} & \text{for } z \leq z_{\text{tr}} \\ H_{\text{tr}} & \text{for } z > z_{\text{tr}} \end{cases} \quad (49)$$

where $\alpha$ and $H_{\text{tr}}$ have values of 0.9 and 0.1, respectively, and $H$ is limited to a maximum value of 0.9. Weisman and Klemp (1982) used values of $\alpha = 0.75$ and $H_{\text{tr}} = 0.25$. The new humidity profile is slightly drier, and the top of the moist boundary layer is slightly lower than from the original formulation.

The concentration of cloud condensation nuclei (CCN) was set at $10^9 \text{ m}^{-3}$, and the droplet dispersion is $\phi = 0.15$, consistent with a continental storm. The rain autoconversion time delay $\tau_{\text{a,o}}$ was set at 5 min. It turns
out, however, that in this case the time delay setting is not important because the high cloud droplet concentration causes the mixing ratio threshold $q_{w,o}$ in (46) to be high enough to prevent rain autoconversion outright. The simulation has many of the characteristics typical of Colorado storms (Dye et al. 1974), which are characterized by a precipitation-free updraft base and cloud droplets that are too small for effective growth by collision–coalescence. The simulated cloud base, however, is more typical of central plains storms (i.e., lower) than of high plains storms (e.g., Dye et al. 1974). The high prescribed CCN concentration guarantees small cloud droplets and effectively turns off autoconversion in the model. All rain originates from the melting of ice particles or shedding from wet collection growth. The results are, therefore, quite different from the simulation of Weisman and Klemp (1984) for the same shear profile. Weisman and Klemp (1984) used a warm rain microphysics scheme that rapidly forms rain via autoconversion in the updraft, and the rain subsequently fell back through the updraft base.

The computational domain was 45 km $\times$ 45 km $\times$ 17.5 km, with constant horizontal grid spacing of 500 m and vertical spacing of 200 m at the surface, stretching to a constant 500 m above 8.75 km. Convection was initiated with a warm spheroid with a central temperature perturbation of 0.9 K. The spheroid radii were 5 km $\times$ 5 km $\times$ 1.5 km. Random fluctuations in the range of $\pm$20% of the local perturbation value were added to the warm bubble to increase entrainment and mixing in the initial thermal. The bubble was not moistened, preserving the dewpoint temperature. Storm longevity is quite sensitive to the sounding and initial conditions. A test simulation using an original Weisman and Klemp (1982) profile without limiting $H$ resulted in substantially stronger and long-lived convection. Another test found that an initial bubble with no random perturbations also resulted in a somewhat stronger final cell.

1) Ice Precipitation Initiation Process

The ice process is most easily seen in the initial updraft cell. Figure 2 shows the evolution of ice crystals, rimed crystals, and low- and medium-density graupel as the initial cell grows. At 17 min the cloud consists of cloud droplets and trace amounts of pristine ice crystals (plates and columns added together). By 18–20 min, cloud ice has rimed sufficiently for small amounts of rimed crystals and graupel to appear. The maximum mixing ratio of rimed crystals stays relatively small because the collection of plentiful supercooled cloud droplets tends to convert rimed crystals rapidly into low-density graupel. Rimming is sufficient to convert some low-density graupel to medium-density graupel, but the low temperature and small droplets prevent any high-density graupel production until later in the storm evolution.

The initial updraft cell contains relatively large mixing ratios (>7 g kg$^{-1}$) of cloud droplets. The high CCN concentration results in droplets that are too small (less than $D_{w,o}$) for effective coalescence, as observed for some Colorado storms (Dye et al. 1974). By 24 min (not shown), and later in the storm (Fig. 5), over 1 g kg$^{-1}$ of supercooled cloud droplets persists at $-35^\circ$C. This relatively high cloud droplet content is consistent with the observations of Rosenfeld and Woodley (2000), who found contents as high as 1.8–4 g m$^{-3}$ at temperatures near $-38^\circ$C.

2) Multicell Evolution

The continental multicell storm has a series of updraft pulses (Fig. 3b) at 20, 30, 37, and 52 min that grow on the northwestern edge of the storm. Each pulse is connected to a relatively steady updraft base at 3–4 km AGL, following the “weak evolution” model of Foote and Frank (1983) based on the Westplains, Colorado, storm. (For comparison, in the “strong evolution” model each updraft has a new base.) At 38 min the third cell is an updraft pulse at 4–6 km (Fig. 4). Precipitation from the first cell has almost reached the ground, while the second cell is decaying. Ice precipitation appears at the top of cell 3 by 44 min (6–9-km altitude), as the main reflectivity region of cell 2 falls below 7 km. Cell 3 reaches maturity by 50 min, and, as in the case studied by Foote and Frank (1983), the updraft base remains free of precipitation (Fig. 4).

The strongest updraft pulse at 52 min is closely followed by a new updraft (and new updraft base) around 62 min. These two updrafts are the first to recycle significant amounts of graupel and meltwater rain from the precipitation shaft back into the updraft, leading to increases in graupel and the formation of hail (Fig. 3c).

The recycling process also results in the appearance of high precipitation content in the updraft (Fig. 5b) before the main cell collapses. A weak left-moving cell develops by about 80 min (Fig. 5a at 26 km north, 16 km east) and dissipates within 15 min.

3) Mature Stage

The multicell storm at its mature stage exhibits all of the 10-ICE hydrometeor types. Figures 5 and 6 show hydrometeor fields and simulated radar reflectivity at 76 min. Cloud ice, aggregates, and cloud droplets are shown in Fig. 5c. The maximum of cloud ice crystal content is at the top of the updraft at temperatures less than $-40^\circ$C, where the cloud droplets freeze homogeneously. The relatively small mixing ratios of aggregates indicate that the mechanism is not very active at the low temperatures ($T < -30^\circ$C) where ice crystals are plentiful but collection and sticking/interlocking efficiencies are low. Low- and medium-density graupel can be seen precipitating out of the storm and melting into rain (Fig. 5b). The lower fall velocity of low-density graupel allows it to be carried farther downstream than...
the medium-density graupel, resulting in an elongated region of precipitation. Rimed ice crystals have rather low mixing ratios (on the order of \(10^{-5} \text{ kg kg}^{-1}\)) and are not shown, but they have a maximum content in the lower half of the anvil where ice crystals and supercooled drops coexist.

The updraft region up to 8 km is shown in greater detail in Fig. 6 (also at 76 min). As suggested by Fig. 5b, meltwater rain is being recycled into the updraft and carried above the freezing level, resulting in frozen drops. Some rain may also appear from shedding from hail or high-density graupel, but the rain at the updraft base (0°C) in Fig. 6a started as meltwater that increased in content via cloud droplet collection. High-density graupel is mainly confined to the updraft core, where it forms from the riming of frozen drops (Fig. 6a) and medium-density graupel. Small hail has appeared as the result of the conversion of the larger graupel and frozen drop particles, and large hail comes from conversion of the upper end of the small hail spectrum (Fig. 6b).
A maritime (Florida) storm simulation shows the warm rain process and subsequent conversion of frozen drops into graupel and hail. The model environment (Fig. 7) is the 1810 UTC sounding from the Tico Airport (TCO) in Florida on 9 August 1991 from the Convection and Precipitation/Electrification (CAPE) field program. The sounding was cooled slightly at the top of the boundary layer to reduce the convective inhibition from 31 to 23 J kg$^{-1}$. The original and modified soundings have CAPE of about 1000 J kg$^{-1}$. The computational domain was 25 km $\times$ 25 km $\times$ 15 km, with constant horizontal and vertical grid spacings of 250 m. The warm bubble (not moistened) had a horizontal radius of 8 km and an average temperature perturbation of 2 K. The only changes made to the microphysics input options from the continental case were the reduction of the CCN concentration to $3 \times 10^8$ m$^{-3}$ and an increase in the dispersion at $\phi = 0.3$, consistent with typical maritime conditions. Both of these changes are related to the aerosols available for cloud formation and can be approximated reliably to discriminate continental and maritime air masses.

The low CCN concentration results in larger droplets and a much lower cloud water mixing ratio threshold for autoconversion to rain [(46)]. The first cloud droplets appear at 6 min after model initiation, and the first raindrops form at 12 min. The warm bubble produces multiple updraft cells due to the random thermal variations. The initial development of one of these cells is shown in Fig. 8. The first traces of rain in this particular cell appear at 16 min and are more apparent by 18 min. Rain mixing ratios increase rapidly by further autoconversion and accretion. By 20 min the rising cloud and rain just reach up to the freezing level.

Further evolution of the cell is shown in Fig. 9. At 22 min the maximum rain mixing ratio exceeds 4 g kg$^{-1}$, and most of the rain is still below the freezing level. Two minutes later (24 min), some of the rain has been carried above the freezing level, and frozen drops coincide with supercooled raindrops. A fraction of the frozen drops are also large enough ($>5$ mm) to be converted to small hail. By 26 min high-density graupel develops from the riming of frozen drops, and low-density riming growth further converts some high-density graupel to medium-density graupel. Small-hail content also increases from accretion of rain and cloud droplets, and some is converted to large hail. The 60-dBZ contour is coincident with the 0.3 g kg$^{-1}$ contour of large hail. At 28 min the small- and large-hail particles have fallen out of the updraft, again reflected by the vertical elongation of the 50-dBZ reflectivity contour. Lower-density riming at a lower temperature ($T \approx -10^\circ$C) results in more medium-density graupel and some low-density graupel near $-20^\circ$C. The peak value of hail at the surface is quite small, with a maximum large hail size (median volume diameter) of about 6 mm and particle concentration of 1.8 m$^{-3}$ (i.e., a few pea-sized hailstones).

The transition from liquid rain to graupel was documented by Bringi et al. (1997) for a storm cell (storm “D”) that occurred near the sounding site on 9 August 1991. The first of three aircraft penetrations at an altitude of 5.5 km ($-6.5^\circ$C) found liquid drops. A second pass 3 min later (1811 UTC) found primarily frozen drops (maximum diameter of 4 mm). Then 4 min after that, the third pass found rough graupel. A second aircraft made parallel penetrations at a lower altitude (4 km or $2.3^\circ$C), and on its 1811 UTC pass it found larger “smooth graupel,” which probably corresponds well with the small-hail category in the model because the maximum observed particle diameter was 7.5 mm. A generally similar transition can be seen in Fig. 9, indicating that the model can produce reasonable results for this type of case. For comparison, a small-hail mix-
ing ratio of 3 g kg$^{-1}$ at 4-km altitude gives a median volume diameter ($D_v$) of about 8 mm and a total concentration of 90 m$^{-3}$. Large hail of 0.3 g kg$^{-1}$ at the same altitude would have $D_v \approx 11$ mm but a concentration of only 3 m$^{-3}$.

4. Discussion and conclusions

The test simulations exhibit some of the reasonable results obtainable with the 10-ICE scheme for storms dominated by different precipitation processes. The continental storm first developed low- and medium-density graupel, and eventually high-density graupel and hail appeared in the storm core. The precipitation distribution at the ground also is strongly influenced by horizontal distribution of ice particles. The storm, in general, follows the “weak” multicell evolution model of Foote and Frank (1983). In the maritime storm, on the other hand, the first large ice particles were frozen drops, which quickly converted to both small hail and high-density graupel. The high-density graupel converted to medium- and low-density graupel as particles
were carried by the updraft to lower temperatures and lower-density rime growth. The results of Gilmore et al. (2004b) and appendix A show that a 3-ICE type of scheme can be tuned toward certain graupel/hail characteristics, but that this approach will leave out a significant range of detail.

A main goal in the development of the 10-ICE microphysical scheme was to have sufficient detail to simulate a variety of convective events with a minimum of parameter tuning. Further motivation was to have a consistent parameterization across storm cases and, thus, have consistent behavior in the electrification parameterizations, which depend directly on the microphysics. The model uses riming history (age) to calculate transitions between graupel and frozen drops categories, providing smoother transitions in particle density and fall speed. Two-moment schemes are another approach to reduce tuning, and they remove the need to set number concentration intercepts. Both approaches add complexity, flexibility, and, presumably, a more accurate representation of the most important mi-

Fig. 5. Mature multicell storm at 76 min. (a) Reflectivity (dBZ) at 4.09-km altitude with updraft contours (5 and 15 m s⁻¹). Dashed line shows the oblique vertical cross section for the other panels. (b) Vertical cross section of reflectivity. Wind vectors are storm relative and shown at every other model point. (c) Contours of cloud ice (0.1, 0.5, 1.0, 2.0 g kg⁻¹), cloud droplets (0.1, 1.0, 2.0, 3.0 g kg⁻¹), and ice aggregates (0.1, 0.2 g kg⁻¹). (d) Contours of rain (0.1, 1.0, 2.0 g kg⁻¹) and graupel [low and medium density (0.5, 1.5, 2.5 g kg⁻¹) and high density (0.2, 0.4, 0.6 g kg⁻¹)]. The gray fill areas in (a) and (b) indicate updraft >10 m s⁻¹.

Fig. 6. Mature multicell storm at 76 min. (a) Contours of rain (0.5, 1.5 g kg⁻¹), frozen drops (0.3, 0.6, 0.9 g kg⁻¹), and high-density graupel (0.3, 0.6 g kg⁻¹). (b) Contours of small hail (0.5, 1.5, 2.5 g kg⁻¹) and large hail (0.3, 0.6, 0.9 g kg⁻¹). The gray fill areas indicate updraft >10 m s⁻¹, and wind vectors are storm relative.
crophysical processes. The detail and complexity of microphysics parameterizations are matters of goals and computational resources. One advantage of the multiple categories in the 10-ICE scheme is that many different types of ice precipitation can coexist in the same grid volume, adding a measure of realism to simulations of complex storms.

A primary use of the 10-ICE scheme is for simulations of electrification and lightning (MacGorman et al. 2001; Mansell et al. 2002). Many laboratory and field studies have implicated interactions between ice particles as the primary source of storm electrification, so a good representation of ice seems to be a prerequisite for realistic modeling of electrification. Helsdon et al. (2001) have already shown some sensitivity of electrification to ice crystal concentration, and our own simulations are showing that predicting concentrations at temperatures of $-15^\circ$ to $0^\circ$C may be especially important. Thus, the prediction of ice concentration has a high priority for future improvements to the 10-ICE model. As mentioned above, prediction of ice crystal mean diameter is also planned and would have an impact on electrification because the charge separation equations depend on crystal size.

Acknowledgments. The authors thank Dr. Conrad Ziegler for reviewing the manuscript and making many useful suggestions. We are also grateful to Drs. Louis Wicker, Donald MacGorman, and Matthew Gilmore for their encouragement, assistance, and suggestions. We also thank three anonymous reviewers for their constructive comments that greatly improved this paper. The authors consider themselves to be co–first authors: Straka was the main architect of the microphysics scheme, and Mansell led the manuscript preparation and generated the examples. Both authors worked to refine the scheme and edit the text. Support for this research was provided under National Science Foundation Grants ATM-0119398, ATM-0003869, ATM-9617318, ATM-9807179, and ATM-0340693. Funding for this research also was provided under NOAA-OU Cooperative Agreement NA17RJ1227.

APPENDIX A

Comparison with 3-ICE Results

Recognizing possible interest in a comparison between the 10-ICE and 3-ICE microphysics schemes, the continental multicell case was rerun using the 3-ICE parameterization. Two runs were made with commonly used settings for the graupel/hail category that emphasize either graupel or hail characteristics. The “graupel” setting used a particle density of 400 kg m$^{-3}$ and slope intercept $n_0 = 4 \times 10^4$ m$^{-4}$ (Rutledge and Hobbs 1984) that emphasizes smaller, lower-density particles. The “hail” setting used a particle density of 900 kg m$^{-3}$ and slope intercept $n_0 = 4 \times 10^4$ m$^{-4}$ (Lin et al. 1983), modeling larger, higher-density particles. The drag coefficient was $C_D = 0.8$ for graupel and $C_D = 0.6$ for hail. One important change to the 3-ICE scheme (GRS04) is that the Ferrier (1994) rain autoconversion scheme has been incorporated so that the warm-rain physics are comparable to the new scheme. The first-order fallout scheme in 3-ICE was also replaced with the sixth-order monotonic scheme used with 10-ICE. The 3-ICE simulations are initialized as for the 10-ICE case, with exactly the same random perturbations in the warm bubble.

Figure A1 shows the same time–height plots as Fig. 3, but for the 3-ICE scheme. The time–height maximum reflectivity and updraft mass flux in the 3-ICE graupel simulation (Figs. A1Aa,b) compare rather closely to the 10-ICE results up to about 60 min. The 3-ICE hail case, on the other hand, generates significantly larger reflectivity early in the simulation (Fig. A1Ba). The sharp increase in reflectivity at 2–4 km results from precipitation flux convergence, as seen by GSR04. The buildup of hail would continue down to lower levels if not for melting into rain, which has lower parameterized reflectivity than hail for the same mixing ratio. The enhanced reflectivity (55–65 dBZ) in the 10-ICE run at 65–85 min (Fig. 3a) is poorly reproduced by the 3-ICE graupel case, which has a peak of only 54.4 dBZ, as compared with 68.7 dBZ for 10-ICE. The 3-ICE hail case produces higher reflectivities throughout the simulation, with 55 dBZ appearing 20 min earlier than for 10-ICE and persisting until at least 130 min. The 10-ICE scheme has reflectivity of 55 dBZ or greater only during 65–85 and 100–105 min.
The 3-ICE graupel case has much more mass in the graupel/hail field than the hail case (Figs. A1Ac, A1Bc). Conversely, the 3-ICE hail case has much more mass in the snowfield (not shown) than 3-ICE graupel. The main cause is the difference between the cloud water collection rate of graupel versus hail (Gilmore et al. 2004b). The larger concentrations of graupel at higher altitudes lead to higher cloud water accretion, leaving less cloud water for snow to collect. Although the graupel has a lower fall speed and accretion rate than hail, this is compensated by a longer time aloft. Gilmore et al. (2004b) show that the accretion rates $q_{h_{ac}}$ and $q_{s_{ac}}$ tend to vary inversely as the graupel/hail category characteristics are varied from large hail to small graupel. A contributing factor is the residence of graupel at higher altitudes than hail, so that it competes more directly with snow for cloud water. Cloud ice fields (not shown) are more similar, where the hail case has slightly higher maximum layer mass than the graupel case (10 versus 6.4 Mg).

The 3-ICE simulations both have higher reflectivity at upper altitudes (Fig. A2, 7–10 km) than the 10-ICE simulation (Fig. 4) because of greater conversion rates of cloud ice to snow. The 3-ICE hail case has significantly larger regions of reflectivity greater than 40 dBZ than the 3-ICE graupel and 10-ICE cases. At 44 min, both 3-ICE simulations have deeper reflectivity regions in the growing cell than the 10-ICE case. Significant precipitation recycling begins sooner in the 3-ICE simulations, as indicated at 44 min by the 10-dBZ reflectivity contour at or below the 0 °C isotherm in the updraft region (20–22-km horizontal distance).

These results with the 3-ICE scheme indicate that tuning the graupel/hail characteristics causes substantial changes in results, as found by Gilmore et al. (2004b). There is no particular setting that can really hope to simulate a complex storm that contains a large variety of ice habits. Extra precipitation categories add more flexibility, or “dynamic range,” in this regard, to generate more realistic details.

**APPENDIX B**

**Deposition (Sublimation) Variables**

We define first the following variables: diffusivity of water vapor $\varphi$ in air; thermal conductivity $K_a$ (Wisner et al. 1972) and kinematic viscosity $\nu$ of air (List 1984); the
Schmidt number \( S_c \); latent heats of evaporation \( L_v \) fusion \( L_f \) [Pruppacher and Klett 1978, their Eqs. (4-85a) and (4-85b)], and sublimation \( L_s \); and the gas constant for water vapor \( R \) as

\[
S_c = \frac{\nu}{\psi},
\]

\[ L_v = 2 \, 500 \, 837 \left( \frac{273.15}{T} \right)^{0.167 + 3.67 \times 10^{-4} T'}, \]

\[ L_f = 333 \, 690 + 2030.6 T - 10.467 T^2, \]

\[ L_s = L_v + L_f, \]

\[ R_v = 461.5 \, \text{J kg}^{-1} \, \text{K}^{-1}. \]
Capacitances for IE distributions ($C_e$), plates ($C_{ip}$), and columns ($C_{ci}$) are given by

$$C_e = \frac{D_c}{2}, \quad (B9)$$

$$C_{ip} = \frac{D_{ip}}{\pi}, \quad \text{and} \quad (B10)$$

$$C_{ci} = D_c e/c, \quad (B11)$$

where

$$c = \ln \left( \frac{1 + e}{1 - e} \right) \quad \text{and} \quad (B12)$$

$$e = \sqrt{1 - \frac{a^2}{D_{ci}^2}}. \quad (B13)$$

Here, $e$ is the eccentricity, $a$ is the column width, and $D_{ci}$ is the column length (Cotton 1972b).

The ventilation term $VENT_x$ for graupel, hail, and frozen drops is

$$VENT_x = 0.78 \Gamma(2) \left( 4g \rho_x \right)^{1/4} \left( \frac{S_c^{1/3} \nu^{-1/2} \Gamma(5 + d_y)}{3 \rho_x} \right)^{1/4}, \quad (B14)$$

and for SA and IR we have

$$VENT_{sa} = 0.78 \Gamma(2) + 0.308 \Gamma(2.75) S_c^{1/3} \nu^{-1/2} \Gamma \left( \frac{5 + d_y}{2} \right) \left( \frac{\rho_x}{\rho_0} \right)^{1/4} \left( \frac{D_{n,sa}}{D_{n,sa}^{1+d_y/2}} \right) \quad \text{and} \quad (B15)$$

$$VENT_{ir} = 0.78 + 0.308 S_c^{1/3} \nu^{-1/2} \left( D_{n,ir} \nabla \right)^{1/2}. \quad (B16)$$

The definition of $VENT_{ci}$ (columns) comes from Hall and Pruppacher (1976):

**Fig. A1.** As in Fig. 3, but for the 3-ICE scheme with the large ice characteristics set either for (left) small graupel ($\rho = 400 \text{ kg m}^{-3}, n_0 = 4 \times 10^6 \text{ m}^{-4}$) or (right) hail ($\rho = 900 \text{ kg m}^{-3}, n_0 = 4 \times 10^4 \text{ m}^{-4}$). Contour intervals are the same for the same categories for easier comparison.
VENT\textsubscript{ci} =
\begin{align*}
1 + 0.14X^2 & \quad \text{for } X = S_i^{1/3} \text{ Re}_{ci}^{1/2} < 1 \quad \text{(B17)} \\
0.86 + 0.28X & \quad \text{for } X \geq 1 \quad \text{(B18)}
\end{align*}

and VENT\textsubscript{ip} (plates) comes from Thorpe and Mason (1966):

VENT\textsubscript{ip} = 0.65 + 0.44S_i^{1/3} \text{ Re}_{ip}^{1/2}, \quad \text{(B19)}

where the Reynolds numbers for columns and plates are (Davis and Auer 1974, converted to SI units)

\[
\text{Re}_{ci} = \frac{1.258 \times 10^4 D_{ci}^{2.331} + 5.662 \times 10^4 D_{ci}^{2.373}}{\nu(0.8241 D_{ci}^{0.042} + 1.70)} \quad \text{(B20)}
\]

and

\[
\text{Re}_{ip} = \frac{1}{\nu} (58.25 D_{ip}^{1.824} + 0.1652 D_{ip}^{1.299}) \quad \text{(B21)}
\]

REFERENCES
Moeng, C.-H., 1984: A large-eddy-simulation model for the study of...


