STAGGERED PRT WITH GROUND CLUTTER FILTERING AND OVERLAID ECHO RECOVERY FOR DUAL POLARIZATION

ALGORITHM DESCRIPTION

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PREFACE

This document extends the previous Staggered PRT algorithm description from July 2009 by including dual polarization sequences and the calculation of polarimetric variables. This algorithm description includes a high-level description with the overall processing logic followed by a detailed explanation of each pre-computation and processing step.

Most of the July 2009 algorithm steps are now repeated for both H- and V-channels calculation. Unlike them, the SACHI filter has been modified preserving the phase value in order to allow the calculation of the polarimetric variables. Now, the output of the SACHI filter produces autocorrelations for both H- and V-channels and also the cross-correlation between them. To ease implementation and reduce ambiguity, most of the steps in the SACHI algorithm are described in algorithmic form. The DC removal ground clutter filter has been retained to operate on those range gates where only long-PRT data is available and ground clutter filtering is needed and now it includes both channels.

As in the July 2009 description, the algorithm is able to handle overlaid echoes, extending the recovery of Doppler moments to the unambiguous range of the long PRT. Moment-specific overlaid power thresholds are used to identify recoverable data and flag unrecoverable Doppler moments. In this version of the algorithm, ground clutter is also assumed to be within the unambiguous range of the short PRT.
**ASSUMPTIONS**

1) The transmission sequence alternates two pulse repetition times (PRT) as: $T_1, T_2, T_1, T_2 \ldots$ for a total of $M$ pulses.

2) The PRT ratio $T_1/T_2 = 2/3$, where $\kappa_m = 2$, $\kappa_n = 3$ and $T_2 - T_1 = T_u$.

3) All range gates are available and there is a perfect alignment of range gates between the two PRTs (i.e., a given range gate represents the same resolution volume in space for every transmitted pulse). Also, the number of range gates for each PRT is: $N_1 = T_1/\tau_s$ and $N_2 = T_2/\tau_s$, where $\tau_s$ is the sampling period.

4) There are no significant echoes beyond the maximum unambiguous range corresponding to $T_2$ ($r_{u2}$).

5) There is no significant ground clutter beyond the maximum unambiguous range corresponding to $T_1$ ($r_{u1}$).

6) The number of staggered PRT samples per range gate ($M$) is even.

7) The algorithm operates on a radial worth of data at a time.

**INPUTS**

1) Dual polarization complex time-series data:

$$
\begin{align*}
V_H(n, m) &= I_H(n, m) + jQ_H(n, m), \\
V_V(n, m) &= I_V(n, m) + jQ_V(n, m),
\end{align*}
$$

where subscripts $H$ and $V$ denote horizontal and vertical polarization, $0 \leq n < N_1$ for even $m$, $0 \leq n < N_2$ for odd $m$ and $0 \leq m < M$. Note that $n$ indexes the range gates and $m$ the sweeps (or pulses).

2) Associated metadata:

- $\lambda$ is the radar wavelength in meters
- $N_H$ is the noise power in linear units for the horizontal channel
- $N_V$ is the noise power in linear units for the vertical channel
- $\text{d}BZ_0$ is the system calibration constant in dB
- $\text{ATMOS}$ is the elevation-dependent atmospheric attenuation in dB/km
- $\Delta R$ is the spacing between range gates in km ($\Delta R = c \tau_s/2$)
- $T_s$ is the signal-to-noise ratio threshold for reflectivity in dB
- $T_v$ is the signal-to-noise ratio threshold for velocity in dB
- $T_w$ is the signal-to-noise ratio threshold for spectrum width in dB
- $T_{ov}$ is the velocity overlaid threshold in dB (Note: recommended value is 0 dB)
- $T_{ow}$ is the spectrum width overlaid threshold in dB (Note: recommended value is 10 dB)

3) Data window:

$d'(m)$, where $0 \leq m < 5M/2$. Note that $d'$ does not need to be normalized or scaled in any way. A tapered data window such as the Blackman window is recommended for best performance of the SACHI ground clutter filter. Otherwise, rectangular window (i.e., no window) should be applied.

4) Ground clutter filter bypass map:

$B(n)$, where $n$ indexes the range bins with the same resolution as the time-series data along a radial, and the map corresponds to the elevation and azimuth of the radial being processed. $B$ is 0 if clutter filtering is required and 1 otherwise. In this algorithm, the clutter map is ignored beyond the unambiguous range corresponding to the short PRT where clutter is assumed not to be present.
OUTPUTS

1) Reflectivity, Doppler velocity, and spectrum width calculated from H channel data:

\[ Z(n) \text{ for } 0 < n < N_2, \]
\[ v(n) \text{ and } w(n) \text{ for } 0 \leq n < N_2. \]

2) Differential reflectivity, differential phase and correlation coefficient calculated from H and V channel data:

\[ Z_{DB}(n) \text{ for } 0 \leq n < N_2, \]
\[ \Phi_{DB}(n) \text{ for } 0 \leq n < N_2, \]
\[ \rho_{HV}(n) \text{ for } 0 \leq n < N_2. \]

3) Signal-to-noise ratio and overlaid censoring flags*:

\[ NS_Z(n), NS_V(n) \text{ and } NS_W(n) \text{ for } 0 < n < N_2, \]
\[ OV_V(n) \text{ and } OV_W(n) \text{ for } 0 \leq n < N_2. \]

* \( NS_Z(n) \) is used for censoring \( Z_{DB}(n), \Phi_{DB}(n) \) and \( \rho_{HV}(n) \).

FUNCTIONS AND CONVENTIONS

1) \(|-|\) – Returns the absolute value of a complex number or the absolute value of each element of a matrix of complex numbers.

2) \(\arg\) – Returns the principal phase angle of the input complex number in radians. The algorithm is written to accommodate this phase in the interval \([0, 2\pi]\) or \([\pi, \pi]\).

3) \(\arg min\) – Returns the index \(k\) to the element in the input vector that has the minimum value.

4) \(\text{diag}\) – Returns a square matrix with the input vector along the principal diagonal (row index = column index) of the matrix and all other elements not on the principal diagonal equal to zero. The number of rows (columns) of the matrix is equal to the number of elements in the vector.

5) \(\text{ceiling}\) – Returns the smallest integer value not less than the input number.

6) \(\text{floor}\) – Returns the largest integer value not greater than the input number.

7) \(\text{round}\) – Returns the nearest integer to the input number.

8) \(\text{max}\) – Returns the maximum value among the input numbers.

9) Italicized names are used to denote scalars (e.g., \(\text{Noise}\)).

10) Bolded names are used to denote vectors or matrices (e.g., \(A\)). Italicized names with indexing in parentheses are used to denote elements of a vector or matrix [e.g., \(A(i,j)\)].


12) \(T\) – Denotes matrix transpose.

13) \(j\) – Denotes the imaginary unit \(\sqrt{-1}\).
HIGH-LEVEL ALGORITHM DESCRIPTION

If first run of SPRT algorithm
  1) Pre-computation of velocity dealiasing rules
  2) Pre-computation of $M$-independent SACHI filter parameters
End

If the number of samples ($M$) changed
  3) Pre-computation of window parameters
  4) Pre-computation of $M$-dependent SACHI filter parameters
End

For each range bin $n$, where $0 \leq n < N_2$
  If $n \geq N_1$
    5) Short-PRT Segment-III Data Reconstruction
End
  If $B(n) = 0$ AND $n < N_1$
    6) SACHI Clutter Filtering (Segment-I/II gate with segment-I/II clutter)
   Else
    7) DC Removal Clutter Filtering (Segment-III gate with segment-I clutter)
   Else
    8) No Clutter Filtering
   End
  9) Power and correlation computations for each PRT
 10) Combined power and cross-correlation computation
End

End

11) Strong point clutter canceling
For each range bin $n$, where $0 \leq n < N_2$
  12) Signal power computation
  13) Reflectivity computation
  14) Velocity computation
  15) Spectrum width computation
  16) Differential reflectivity computation
  17) Differential phase computation
  18) Cross-correlation coefficient computation
  19) Determination of significant returns for reflectivity and polarimetric variables
  20) Determination of significant returns for velocity
  21) Determination of significant returns for spectrum width
End

For each range bin $n$, where $0 \leq n < N_2$
  22) Determination of overlaid returns for velocity and spectrum width
End
STEP-BY-STEP ALGORITHM DESCRIPTION

1) Pre-computation of velocity dealiasing rules

This method is described in the paper “Design, Implementation, and Demonstration of a Staggered PRT Algorithm for the WSR-88D” by Torres et al. (2004). Herein, $VDA_c$ are the normalized velocity difference transfer function (VDTF) constant values and $VDA_p$ are the normalized number of Nyquist co-intervals for dealiasing.

A set of velocity dealiasing rules is pre-computed at the initiation of the SPRT algorithm as follows:

(Compute type-I and II positive VDTF discontinuity points. $\kappa_m$ and $\kappa_n$ are the integers in the PRT ratio)

$p = 0$

While $2p + 1 < \kappa_m$

\[ D_1 (p) = \frac{2p + 1}{\kappa_m} \]

\[ TYPE_1 (p) = 1 \]

$p = p + 1$

End

$q = 0$

While $2q + 1 < \kappa_n$

\[ D_2 (q) = \frac{2q + 1}{\kappa_n} \]

\[ TYPE_2 (q) = 2 \]

$q = q + 1$

End

(Create $TYPE$ by combining and sorting both sets of discontinuity points)

Concatenate $D_1$ and $D_2$ to create $D$ with $p + q$ elements.

Concatenate $TYPE_1$ and $TYPE_2$ to create $TYPE$ with $p + q$ elements.

Sort $TYPE$ in a “slave” mode using $D$ as the “master”.

(Compute VDTF constants and dealiasing factors for non-negative discontinuity points)

$VDA_c (p + q) = 0$

$VDA_p (p + q) = 0$

For $0 \leq k < p + q$

If $TYPE (k) = 1$

\[ VDA_c (p + q + k + 1) = VDA_c (p + q + k) - \frac{2}{\kappa_m} \]

\[ VDA_p (p + q + k + 1) = VDA_p (p + q + k) + \frac{1}{\kappa_m} \]

Else

\[ VDA_c (p + q + k + 1) = VDA_c (p + q + k) + \frac{2}{\kappa_n} \]

\[ VDA_p (p + q + k + 1) = VDA_p (p + q + k) \]

End

End

(Compute VDTF constants and dealiasing factors for negative discontinuity points)

For $-(p + q) \leq k < 0$

\[ VDA_c (p + q + k) = -VDA_c (p + q - k) \]

\[ VDA_p (p + q + k) = -VDA_p (p + q - k) \]

End

(Note that since the PRT ratio does not change, these vectors can be hard-coded in a real-time implementation of the SPRT algorithm.)
Pre-computation of $M$-independent SACHI filter parameters

This method is described in NSSL Signal Design and Processing Techniques for WSR-88D Ambiguity Resolution (Report 3, Report 9 and Report 11). The SACHI filter parameters could be pre-computed at the initiation of the SPRT algorithm as follows:

(1) **Create 5-by-5 convolution matrix, $C_r$**

$$
C_r = 
\begin{bmatrix}
C(0) & C(4) & C(3) & C(2) & C(1) \\
C(1) & C(0) & C(4) & C(3) & C(2) \\
C(2) & C(1) & C(0) & C(4) & C(3) \\
C(3) & C(2) & C(1) & C(0) & C(4) \\
C(4) & C(3) & C(2) & C(1) & C(0)
\end{bmatrix}
$$

where $C(k) = \frac{1}{\sqrt{10}} \sum_{n=0}^{4} c(n) \exp(-j2\pi nk/5)$; for $0 \leq k < 5$ and $c = [1, 0, 1, 0, 0]$, and $C_{r,k}$ is the $k$-th column of $C_r$.

(2) **Calculate magnitude deconvolution matrix, $C_{md}$**

(Note: The following formulas are written in matrix algebra notation with the conventions described above)

$$
C_{md} = |C_r|^{-1} =
\begin{bmatrix}
C_{md,1} & C_{md,2} & C_{md,3} & C_{md,4} & C_{md,5} \\
-4.6281 & -2.0697 & 4.6281 & 4.6281 & -2.0697 \\
-2.0697 & -4.6281 & -2.0697 & 4.6281 & 4.6281 \\
4.6281 & -2.0697 & -4.6281 & -2.0697 & 4.6281 \\
4.6281 & 4.6281 & -2.0697 & -4.6281 & -2.0697 \\
-2.0697 & 4.6281 & 4.6281 & -2.0697 & -4.6281
\end{bmatrix}
$$

where $C_{md,k}$ is the $k$-th row of $C_{md}$.

(3) **Calculate matrices $C_{R1}$ and $C_{R2}$ using 1st and 5th columns of $C_r$**

$$
C_{R1} = C_{r,1} C_r^T =
\begin{bmatrix}
0.4 & 0.0382 + j0.1176 & 0.2618 - j0.1902 & 0.2618 + j0.1902 & 0.0382 - j0.1176 \\
0.0382 - j0.1176 & 0.0382 & -0.0309 - j0.0951 & 0.0809 - j0.0588 & -0.0309 - j0.0225 \\
0.2618 + j0.1902 & -0.0309 + j0.0951 & 0.2681 & 0.0809 + j0.2490 & 0.0809 - j0.0588 \\
0.2618 - j0.1902 & 0.0809 + j0.0588 & 0.0809 - j0.2490 & 0.2681 & -0.0309 - j0.0951 \\
0.0382 + j0.1176 & -0.0309 + j0.0225 & 0.0809 + j0.0588 & -0.0309 + j0.0951 & 0.0382
\end{bmatrix}
$$

$$
C_{R2} = C_{r,5} C_r^T =
\begin{bmatrix}
0.0382 & -0.0309 - j0.0951 & 0.0809 - j0.0588 & -0.0309 - j0.0225 & 0.0382 - j0.1176 \\
-0.0309 + j0.0951 & 0.2618 & 0.0809 + j0.2490 & 0.0809 - j0.0588 & 0.2618 + j0.1902 \\
0.0809 + j0.0588 & 0.0809 - j0.2490 & 0.2681 & -0.0309 - j0.0951 & 0.2618 - j0.1902 \\
-0.0309 + j0.0225 & 0.0809 + j0.0588 & -0.0309 + j0.0951 & 0.0382 & 0.0382 + j0.1176 \\
0.0382 + j0.1176 & 0.2618 - j0.1902 & 0.2618 + j0.1902 & 0.0382 - j0.1176 & 0.4
\end{bmatrix}
$$

where $^T$ stands for the matrix conjugate transpose (a.k.a. Hermitian) operation.

(4) **Calculate the correction coefficients $\xi_2$ and $\xi_3$ for correction vector $X$**

$$
\xi_k = \frac{1}{C_{md,k} \left| C_{r,k} - (C_r^T C_{r,k}) C_{r,k} \right|}, k = 2, 3.
$$

$\xi_2 = 1.1056$ and $\xi_3 = 1.7889$. 

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(Note: since the PRT ratio does not change, these matrices and coefficients can be hard-coded in a real-time implementation of the SPRT algorithm. The numbers provided here are for reference purposes only; the highest precision available is recommended for hard-coding these numbers.)

3) Pre-computation of window parameters

(Calculate the extended number of coefficients)

\[ M_x = \frac{5M}{2} \]

(Calculate the number of pulse pairs)

\[ M_p = \frac{M}{2} \]

(Calculate normalized window \(d\) for un-normalized window function \(d'\) with \(M_x\) points)

\[ d(m) = d'(m) \left( \frac{1}{M_x} \sum_{m=0}^{M_x-1} [d'(m)]^2 \right)^{-1} ; \ 0 \leq m < M_x. \]

(Calculate window correction factor for lag-1)

\[ d_c = \frac{1}{M_x} \sum_{m=0}^{M_x-2} d(m)d(m+1) \]

4) Pre-computation of \(M\)-dependent SACHI filter parameters

(Compute correction vector, \(X\))

For \(0 \leq k < \text{ceiling}(M_p/2)\)

\[ X(k) = 1 \]

End

For \(\text{ceiling}(M_p/2) \leq k < \text{ceiling}(M_p/2) + M_p\)

\[ X(k) = \xi_2 \]

End

For \(\text{ceiling}(M_p/2) + M_p \leq k < \text{ceiling}(M_p/2) + 3M_p\)

\[ X(k) = \xi_3 \]

End

For \(\text{ceiling}(M_p/2) + 3M_p \leq k < \text{ceiling}(M_p/2) + 4M_p\)

\[ X(k) = \xi_2 \]

End

For \(\text{ceiling}(M_p/2) + 4M_p \leq k < M_x\)

\[ X(k) = 1 \]

End
5) Short-PRT Segment-III Data Reconstruction

Long-PRT Segment-I data is used as a proxy for short-PRT segment-III data

For $0 \leq m < M_p$

\[
V_H(n, 2m) = V_H(n - N_1, 2m + 1)
\]

\[
V_V(n, 2m) = V_V(n - N_1, 2m + 1)
\]

End

6) SACHI Clutter Filtering

The SACHI filter algorithm is used when clutter filtering is required inside the maximum unambiguous range corresponding to $T_1 (r_{st})$.

(Form derived time series, $V_{id}$ and $V_{vd}$ from input time series $V_H$ and $V_V$)

For $0 \leq m < M_p$

\[
V_{id}(5m) = V_H(n, 2m)
\]

\[
V_{id}(5m + 1) = 0
\]

\[
V_{id}(5m + 2) = V_H(n, 2m + 1)
\]

\[
V_{id}(5m + 3) = 0
\]

\[
V_{id}(5m + 4) = 0
\]

\[
V_{vd}(5m) = V_V(n, 2m)
\]

\[
V_{vd}(5m + 1) = 0
\]

\[
V_{vd}(5m + 2) = V_V(n, 2m + 1)
\]

\[
V_{vd}(5m + 3) = 0
\]

\[
V_{vd}(5m + 4) = 0
\]

End

(Compute DFT of windowed extended time series power compensated for added zeroes)

\[
F_H(k) = \left( \frac{\sqrt{2}}{M_x} \right) \left( \frac{1}{M_x} \sum_{m=0}^{M_x-1} V_{id}(m)d(m) \exp(-j2\pi km / M_x) \right); \quad k = 0, 1, \ldots, M_x - 1.
\]

\[
F_V(k) = \left( \frac{\sqrt{2}}{M_x} \right) \left( \frac{1}{M_x} \sum_{m=0}^{M_x-1} V_{vd}(m)d(m) \exp(-j2\pi km / M_x) \right); \quad k = 0, 1, \ldots, M_x - 1.
\]

(Determine clutter filter width parameter, $q$)

(Use GMAP to return the number of coefficients identified as clutter, $GMAP_{Hcoef}$ and $GMAP_{Vcoeff}$. Pass to GMAP the $5^{th}$ of the Doppler spectrum containing the main clutter replica; i.e., $|F_{H,V}(0)|^2$, $|F_{H,V}[\text{ceiling}(M_p/2) - 1]|^2$, $|F_{H,V}[M_x - \text{floor}(M_p/2)]|^2$, $|F_{H,V}(M_x - 1)|^2$; initialize GMAP for spectra with $v_5/5$, and get the number of coefficients identified as clutter to estimate $q$ for both $H$ and $V$ channels)

\[
q_H = \text{floor} \left( \frac{(GMAP_{Hcoef} + 1)}{2} \right)
\]

\[
q_V = \text{floor} \left( \frac{(GMAP_{Vcoeff} + 1)}{2} \right)
\]

(Use the largest $q$ to create the clutter filter vectors for polarimetric variable calculation)

\[
q' = \max(q_H, q_V)
\]
(Create clutter filter vectors $I_{f1}', I_{f2}')$

For $0 \leq k < M_p$

If $k < q'$

$\ I_{f1}'(k) = 1$

$\ I_{f2}'(k) = 0$

ElseIf $k \leq M_p - q'$

$\ I_{f1}'(k) = 0$

$\ I_{f2}'(k) = 0$

Else

$\ I_{f1}'(k) = 0$

$\ I_{f2}'(k) = 1$

End

End

(Row-wise re-arrange $F_H$ and $F_V$ into 5-by-$M_p$ matrices, $F_{H_r}$ and $F_{V_r}$)

For $0 \leq k < M_p$

$\ F_{H_r}(0, k) = F_H(k)$

$\ F_{H_r}(1, k) = F_H(k + M_p)$

$\ F_{H_r}(2, k) = F_H(k + 2M_p)$

$\ F_{H_r}(3, k) = F_H(k + 3M_p)$

$\ F_{H_r}(4, k) = F_H(k + 4M_p)$

$\ F_{V_r}(0, k) = F_V(k)$

$\ F_{V_r}(1, k) = F_V(k + M_p)$

$\ F_{V_r}(2, k) = F_V(k + 2M_p)$

$\ F_{V_r}(3, k) = F_V(k + 3M_p)$

$\ F_{V_r}(4, k) = F_V(k + 4M_p)$

End

(Compute the clutter filtered spectrum matrices, $F_{Hf}$ and $F_{Vf}$)

(Note: The following formulas are written in matrix algebra notation. Complex-matrix multiplications can be implemented using four real-matrix multiplications as: $AB = (A_r + jA_i)(B_r + jB_i) = (A_rB_r - A_iB_i) + j(A_rB_i + A_iB_r)$)

$\ F_{Hf} = F_{H_r} - C_{fl}F_{H_r}\ \text{diag}(I_{f1}') - C_{f2}F_{H_r}\ \text{diag}(I_{f2}')$

$\ F_{Vf} = F_{V_r} - C_{fl}F_{V_r}\ \text{diag}(I_{f1}') - C_{f2}F_{V_r}\ \text{diag}(I_{f2}')$

(Row-wise unfold $F_{Hf}$ and $F_{Vf}$ into $F_{Hdf}$ and $F_{Vdf}$)

For $0 \leq k < M_p$

$\ F_{Hdf}(k) = F_{Hf}(0, k)$

$\ F_{Hdf}(k + M_p) = F_{Hf}(1, k)$

$\ F_{Hdf}(k + 2M_p) = F_{Hf}(2, k)$

$\ F_{Hdf}(k + 3M_p) = F_{Hf}(3, k)$

$\ F_{Hdf}(k + 4M_p) = F_{Hf}(4, k)$

$\ F_{Vdf}(k) = F_{Vf}(0, k)$

$\ F_{Vdf}(k + M_p) = F_{Vf}(1, k)$
\[ F_{Vdf}(k + 2M_p) = F_{Vf}(2, k) \]
\[ F_{Vdf}(k + 3M_p) = F_{Vf}(3, k) \]
\[ F_{Vdf}(k + 4M_p) = F_{Vf}(4, k) \]

End

(Compute mean power for both channels, \( P_{P'} \) and \( P_V \), and cross-correlation at lag 0, \( R_{HV}(0) \))

\[
P_{P'}(n) = \sum_{k=0}^{M_p-1} |F_{lag}(k)|^2
\]
\[
P_V(n) = \sum_{k=0}^{M_p-1} |F_{Vdf}(k)|^2
\]
\[
R_{HV}(n) = \sum_{k=0}^{M_p-1} F_{lag}^*(k)F_{Vdf}(k)
\]

(Proceed with the conventional SACHI Clutter Filtering, only H-channel data)

\( q = q_H \)

If \( q < q' \)

(Create clutter filter vectors \( I_f \))

For 0 \( \leq k < M_p \)

If \( k < q \)

\( I_f_1(k) = 1 \)
\( I_f_2(k) = 0 \)

ElseIf \( k \leq M_p - q \)

\( I_f_1(k) = 0 \)
\( I_f_2(k) = 0 \)

Else

\( I_f_1(k) = 0 \)
\( I_f_2(k) = 1 \)

End

End

\( F_f = F_{hr} - C_{\alpha}F_{hr} \text{ diag } (I_f_1) - C_{\alpha}F_{hr} \text{ diag } (I_f_2) \)

Else

\( F_f = F_{hr} \)

End

(Create clutter filter vectors, \( I_1 \) and \( I_2 \))

For 0 \( \leq k < M_p \)

If \( k < q \)

\( I_1(k) = 0 \)
\( I_1(k + M_p) = 0 \)
\( I_1(k + 2M_p) = 0 \)

Else
ElseIf $k \leq M_p - q$
    $I_1 (k) = 1$
    $I_1 (k + M_p) = 1$
    $I_1 (k + 2M_p) = 1$
    $I_1 (k + 3M_p) = 1$
    $I_1 (k + 4M_p) = 1$
    $I_2 (k) = 0$
    $I_2 (k + M_p) = 0$
    $I_2 (k + 2M_p) = 0$
    $I_2 (k + 3M_p) = 0$
    $I_2 (k + 4M_p) = 0$
Else
    $I_1 (k) = 0$
    $I_1 (k + M_p) = 0$
    $I_1 (k + 2M_p) = 0$
    $I_1 (k + 3M_p) = 0$
    $I_1 (k + 4M_p) = 0$
    $I_2 (k) = 1$
    $I_2 (k + M_p) = 1$
    $I_2 (k + 2M_p) = 1$
    $I_2 (k + 3M_p) = 1$
    $I_2 (k + 4M_p) = 1$
End

(Magnitude deconvolved matrix, $F_d$)

$F_d = C_m |F_r|$  

(Row-wise unfold $F_d$ into $F_{d\theta}$)

For $0 \leq k < M_p$
    $F_{d\theta}(k) = F_d(0, k)$
    $F_{d\theta}(k + M_p) = F_d(1, k)$
    $F_{d\theta}(k + 2M_p) = F_d(2, k)$
    $F_{d\theta}(k + 3M_p) = F_d(3, k)$
    $F_{d\theta}(k + 4M_p) = F_d(4, k)$

End

(Compute the lag-1 autocorrelation, $R_{d\theta}$)

$R_{d\theta} = \frac{1}{d_f} \sum_{k=0}^{M_p-1} |F_{d\theta}(k)|^2 \exp(j2\pi k / M_s)$

(Compute vector $I$, with $M/2$ ones centered on $\arg(R_{d\theta})$)
(Round to the nearest spectral coefficient. Choose symmetric window of coefficients around it)

\[ k_{0df} = \text{round} \left( \frac{M \cdot \arg \left( R_{1df} \right)}{2\pi} \right) \]

If \( k_{0df} < 0 \)
\[ k_{0df} = k_{0df} + M_x \]
End
If \( k_{0df} \geq M_x \)
\[ k_{0df} = k_{0df} - M_x \]
End
\[ k_{1df} = k_{0df} - \text{floor}(M / 4) \]
If \( k_{1df} < 0 \)
\[ k_{1df} = k_{1df} + M_x \]
End
\[ k_{2df} = k_{0df} + \text{ceiling}(M / 4) - 1 \]
If \( k_{2df} \geq M_x \)
\[ k_{2df} = k_{2df} - M_x \]
End

\((k_{0df} \text{ is the coefficient corresponding to } \arg (R_{1df}). \ k_{1df} \text{ and } k_{2df} \text{ specify the extent of } M_p \text{ spectral coefficients centered on the mean velocity. If } k_{1df} < k_{2df}, \text{ the ones span from } k_{1df} \text{ to } k_{2df}; \text{ otherwise, the ones will span from } k_{1df} \text{ to } M_x - 1, \text{ and } 0 \text{ to } k_{2df})\)

If \( k_{1df} < k_{2df} \)
For \( 0 \leq k < M_x \)
\[ I_v(k) = \begin{cases} 0 & \text{if } k < k_{1df} \text{ OR } k > k_{2df} \\ 1 & \text{else} \end{cases} \]
End
Else
For \( 0 \leq k < M_x \)
\[ I_v(k) = \begin{cases} 0 & \text{if } k < k_{1df} \text{ AND } k > k_{2df} \\ 1 & \text{else} \end{cases} \]
End
End

(Interpolate the elements for the region around zero velocity in \( F_{df} \) with linearly interpolated values from \( S_1 \) and \( S_2 \))

If \( q > 0 \)
\[ S_1 = \left| F_{df}(q) \right|^2 \]
\[ S_2 = \left| F_{df}(M_x - q) \right|^2 \]
For \( 0 \leq k < M_x \)
\[ F_v(k) = \begin{cases} S_2 + (S_1 - S_2) \left( q + k \right) / 2q & \text{if } k < q \\ S_2 + (S_1 - S_2) \left( q - M_x + k \right) / 2q & \text{else} \end{cases} \]
Else
\[ F_v(k) = \begin{cases} S_2 + (S_1 - S_2) \left( q + k - M_x \right) / 2q & \text{if } k > M_x - q \\ S_2 + (S_1 - S_2) \left( q - M_x + k \right) / 2q & \text{else} \end{cases} \]

Else
\[ F_i(k) = F_{\text{off}}(k) \]

End

End

Else

(Don’t interpolate if not needed)

For \(0 \leq k < M_x\)

\[ F_i(k) = F_{\text{off}}(k) \]

End

End

(Compute the corrected spectrum, \(F_c\))

For \(0 \leq k < M_x\)

\[ F_c(k) = F_i(k) I_1(k) + F_i(k) I_2(k) I_v(k) X(k) \]

End

(Compute vector \(I_c\) with ones where there’s a non-zero spectral component in vector \(F_c\))

For \(0 \leq k < M_x\)

\[ I_c(k) = I_1(k) + I_2(k) I_v(k) \]

End

(Compute the mean power, \(P_c\), and autocorrelation at lag \(T_m\), \(R_{1c}\), using \(F_c\))

\[ P_c = \sum_{k=0}^{M_x-1} \left| F_c(k) \right|^2 \]

\[ R_{1c} = \frac{1}{d_x} \sum_{k=0}^{M_x-1} \left| F_c(k) \right|^2 \exp(j2\pi k/M_x) \]

(Retain only \(M\) coefficients centered on velocity based on \(R_{1c}\) and delete the rest from \(F_c\) and \(I_c\))

\[ k_{0c} = \text{round} \left[ \frac{M_x \cdot \text{arg}(R_{1c})}{2\pi} \right] \]

If \(k_{0c} < 0\)

\[ k_{0c} = k_{0c} + M_x \]

End

If \(k_{0c} \geq M_x\)

\[ k_{0c} = k_{0c} - M_x \]

End

\[ k_{1c} = k_{0c} - M_p \]

If \(k_{1c} < 0\)

\[ k_{1c} = k_{1c} + M_x \]

End

\[ k_{2c} = k_{0c} + M_p - 1 \]

If \(k_{2c} \geq M_x\)

\[ k_{2c} = k_{2c} - M_x \]

End

If \(k_{1c} < k_{2c}\)

For \(0 \leq k < M_x\)

If \(k < k_{1c}\) OR \(k > k_{2c}\)

\[ F_m(k) = 0 \]

\[ I_m(k) = 0 \]

Else
\[ F_m(k) = F_c(k) \]
\[ I_m(k) = I_c(k) \]
End
Else
For \( 0 \leq k < M_x \)
If \( k < k_{1c} \) AND \( k > k_{2c} \)
\[ F_m(k) = 0 \]
\[ I_m(k) = 0 \]
Else
\[ F_m(k) = F_c(k) \]
\[ I_m(k) = I_c(k) \]
End
End
End

(Compute the modified mean power, \( P_m \), and autocorrelation at lag \( T_w \) \( R_{1m} \) using \( F_m \))
\[ P_m = \sum_{k=0}^{M_x-1} |F_m(k)|^2 \]
\[ R_{1m} = \frac{1}{d_x} \sum_{k=0}^{M_x-1} |F_m(k)|^2 \exp \left( j 2\pi k / M_x \right) \]

(Compute noise correction factors)
\[ N_c = \frac{1}{M_x} \sum_{k=0}^{M_x-1} I_c(k) \]
\[ N_m = \frac{1}{M_x} \sum_{k=0}^{M_x-1} I_m(k) \]

(Compute overlaid power correction if in segment I)
If \( n < N_2 - N_1 \)
\[ S_{ov} = \frac{1}{2} \left[ \sum_{m=0}^{M_x-1} \left| H_{m} \left( n + N_1, 2m + 1 \right) \right|^2 - \text{Noise} \right] \]
If \( S_{ov} < 0 \)
\[ S_{ov} = 0 \]
Else
\[ S_{ov} = 0 \]
End

(Correct powers to remove overlaid contamination adjusted for each spectrum)
\[ P_m = P_m - N_m S_{ov} \]
If \( P_m < 0 \)
\[ P_m = 0 \]
End
\[ P_c = P_c - N_c S_{ov} \]
If \( P_c < 0 \)
\[ P_c = 0 \]
End
(Compute spectrum width power ratio adjustment)
\[ S_m = P_m - N_m \text{Noise} \]
If \( S_m < 0 \)
\[ S_m = 0 \]
End
If \( S_m > 0 \)
\[ P_{adj} = \frac{|R_n|}{S_m} \]
Else
\[ P_{adj} = 0 \]
End

(Compute signal power)
\[ S_c = P_c - N_c \text{Noise} \]
If \( S_c < 0 \)
\[ S_c = 0 \]
End

(Compute short PRT autocorrelation at lag \( T_1 \))
\[ R_{H1}(n) = S_c \cdot P_{adj} \exp \left[ j2\arg \left( R_{uc} \right) \right] \]

(Compute long PRT autocorrelation at lag \( T_2 \))
\[ R_{H2}(n) = S_c \cdot P_{adj} \exp \left[ j3\arg \left( R_{uc} \right) \right] \]

(Adjust signal power to include noise)
\[ P_i(n) = S_c + \text{Noise} \]

(Note that the outputs of SACHI are \( P'_i(n), P_v(n), R_{H1}(n), P_{H}(n), R_{H2}(n) \) and \( R_{H2}(n) \))

7) DC Removal Clutter Filtering (Segment-III gate with segment-I clutter)

This DC Removal clutter filtering algorithm removes the mean (DC) component of the short-PRT segment-III gates in those locations where the site-dependent clutter filter bypass map \( B \) indicates the need for clutter within segment I.

(Calculate the mean of the even pulses.)
\[ V_{Hn} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{H}(n,2m) \]
\[ V_{Vn} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{V}(n,2m) \]

(Subtract mean from even pulses.)
For \( 0 \leq m < M_p \)
\[ V_{HF}(2m) = V_{H}(n,2m) - V_{Hn} \]
\[ V_{HF}(2m+1) = V_{H}(n,2m+1) \]
\[ V_{VF}(2m) = V_{V}(n,2m) - V_{Vn} \]
\[ V_{VF}(2m+1) = V_{V}(n,2m+1) \]
8) No Clutter Filtering

For $0 \leq m < M$

$V_{HF}(m) = V_H(n, m)$

$V_{VF}(m) = V_V(n, m)$

End

9) Power and correlation computations for each PRT

If $n < N_1$

(Compute power from even pulses, if available)

$P_{HF1} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{HF}^{*}(2m)|^2$

$P_{VF1} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{VF}^{*}(2m)|^2$

(Compute cross-correlation from even pulses, if available)

$R_{HF1}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^{*}(2m)V_{VF}^{*}(2m)$

End

(Compute power from odd pulses)

$P_{HF2} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{HF}^{*}(2m+1)|^2$

$P_{VF2} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{VF}^{*}(2m+1)|^2$

(Compute cross-correlation from odd pulses)

$R_{HF2}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^{*}(2m+1)V_{VF}^{*}(2m+1)$

(Compute lag-1 correlations from all pulses from H channel)

$R_{HF1}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^{*}(2m)V_{HF}^{*}(2m+1)$

$R_{HF2}(n) = \frac{1}{M_p - 1} \sum_{m=0}^{M_p-2} V_{HF}^{*}(2m+1)V_{HF}^{*}(2m+2)$

10) Combined power and cross-correlation computation

To use as much information as possible, data are extracted from the two power arrays with different rules for each of the three segments depicted in Figure 1. For segment I, data are extracted only from $P_1$, since $P_2$ may be contaminated on those range bins with overlaid powers. An average of $P_1$ and $P_2$ is extracted for segment II, given that both power vectors are “clean” there. Finally, segment III data are obtained from $P_2$. In algorithmic form:
If \( n < N_2 - N_1 \)
  (Segment I)
  \( P_H(n) = P_{H1} \)
  \( P_V(n) = P_{V1} \)
ElseIf \( n < N_1 \)
  (Segment II)
  \( P_H(n) = \frac{1}{2} (P_{H1} + P_{H2}) \)
  \( P_V(n) = \frac{1}{2} (P_{V1} + P_{V2}) \)
Else
  (Segment III)
  \( P_H(n) = P_{H2} \)
  \( P_V(n) = P_{V2} \)
End
\( P'_H(n) = P_H \)

---

Fig. 1. Signal powers in the staggered PRT algorithm. Roman numerals indicate segment numbers.

The same rules apply for the cross-correlation computation.

If \( P_H(n) < N_H \)
  (Segment I)
  \( R_{H'V}(n) = R_{H'V1} \)
ElseIf \( n < N_1 \)
  (Segment II)
  \( R_{H'V}(n) = \frac{1}{2} (R_{H'V1} + R_{H'V2}) \)
Else
  (Segment III)
  \( R_{H'V}(n) = R_{H'V2} \)
End

11) Strong point clutter canceling

Processing is as in the current system. Strong-point clutter canceling is applied to \( P_H, P'_H, R_{H1} \) and \( R_{H2} \) based on radial power continuity in \( P_H \). For the remainder of the algorithm it is assumed that the outputs of this step are \( P_H, P'_H, R_{H1} \) and \( R_{H2} \).

12) Signal power computation

If \( P_H(n) < N_H \)
  \( S_H = 0 \)
Else
  \( S_H = P_H(n) - N_H \)
End

If \( P'_H(n) < N_H \)
13) Reflectivity computation

(Range in km)
\[ R = n\Delta R + \Delta R/2 \]

(Reflectivity in dBZ. \( \log_{10} \) is the base-10 logarithm)
If \( S_H > 0 \)
\[ Z(n) = 10\log_{10} (S_H) + dBZ0 + R\, ATOMS + 20\log_{10} (R) - 10\log_{10} (N_H), \]
Else
\[ Z(n) \text{ should be set to the smallest possible reflectivity value} \]
End

14) Velocity computation

(Compute Doppler velocities for each PRT using the corresponding correlation estimates)
\[ v_1 = -\frac{\lambda}{4\pi T_1} \arg[R_{H1}(n)] \]
\[ v_2 = -\frac{\lambda}{4\pi T_2} \arg[R_{H2}(n)] \]

(Compute extended Nyquist velocity)
\[ v_u = \frac{\lambda}{2T_1} \]
(Dealias velocity using pre-computed rules)
\[ l = \arg \min_k |v_1 - v_2 - VDA_c (k)v_a| \]
\[ v(n) = v_1 + 2v_u \cdot VDA_p (l) \]

(Prevent dealiased velocities outside of the extended Nyquist co-interval)
If \( v(n) > v_u \)
\[ v(n) = v(n) - 2v_u \]
End
If \( v(n) < -v_u \)
\[ v(n) = v(n) + 2v_u \]
End

15) Spectrum width computation
The spectrum width estimator corresponds to the algorithm implemented in the legacy WSR-88D signal processor.

If \( S_H = 0 \) OR \( |R_{HH}(n)| = 0 \)

\[ w(n) = \frac{\lambda}{4\sqrt{3T_i}} \]

ElseIf \( S_H < |R_{HH}(n)| \)

\( (Insert \ spectrum \ width \ of \ a \ constant) \)

\[ w(n) = 0 \]

Else

\( (Spectrum \ width \ computation. \ \ln \ is \ the \ natural \ logarithm) \)

\[ w(n) = \frac{\lambda}{2\sqrt{2\pi T_i}} \sqrt{\ln \left( \frac{S}{|R_{HH}(n)|} \right)} \]

If \( w(n) > \frac{\lambda}{4\sqrt{3T_i}} \)

\[ w(n) = \frac{\lambda}{4\sqrt{3T_i}} \]

End

End

16) Differential reflectivity computation

If \( S'_H > 0 \) AND \( S'_V > 0 \)

\[ Z_{DRA}(n) = 10 \log_{10} \frac{S'_H}{S'_V} \]

ElseIf \( S'_H = 0 \)

\( Z_{DRA}(n) \) should be set to the smallest possible value

ElseIf \( S'_V = 0 \)

\( Z_{DRA}(n) \) should be set to the highest possible value

End

17) Differential phase computation

\[ \Phi_{DPA}(n) = \arg[R_{HV}(n)] \]

18) Cross-correlation coefficient computation

If \( S'_H > 0 \) AND \( S'_V > 0 \)
\[ \rho_{hv}(n) = \frac{|R_{hv}(n)|}{\sqrt{S_H S_V}} \]

Else

\[ \rho_{hv}(n) = 0 \]

End

19) Determination of significant returns for reflectivity and polarimetric variables

The non-significant return indicator array \((NS_Z)\) is a binary array where 0 indicates “significant” and 1 indicates “non-significant”. This array is also used for \(Z_{Dv}(n), \Phi_{Dv}(n)\) and \(\rho_{hv}(n)\).

If \( S_H < N_H \cdot 10^{0.175} \)

\[ NS_A(n) = 1 \]

Else

\[ NS_A(n) = 0 \]

End

20) Determination of significant returns for velocity

The non-significant return indicator array \((NS_V)\) is a binary array where 0 indicates “significant” and 1 indicates “non-significant”

If \( S_H < N_H \cdot 10^{0.175} \)

\[ NS_V(n) = 1 \]

Else

\[ NS_V(n) = 0 \]

End

21) Determination of significant returns for spectrum width

The non-significant return indicator array \((NS_W)\) is a binary array where 0 indicates “significant” and 1 indicates “non-significant”

If \( S_H < N_H \cdot 10^{0.175} \)

\[ NS_W(n) = 1 \]

Else

\[ NS_W(n) = 0 \]

End

22) Determination of overlaid returns for velocity and spectrum width

Censoring of velocity and spectrum width data is only necessary in segments I and III. This is done by analyzing \( P \) in segment I \((P_1)\) and \( P \) in segment III \((P_2)\) (see Fig. 1). The idea is to determine whether second trip signals mask first trip signals and vice versa. While such overlaid echoes appear in every other pulse and do not bias velocity estimates at those range locations, overlaid powers act as noise. Therefore, when overlaid powers are above a preset fraction of their non-overlaid counterparts, the corresponding velocity and spectrum width estimates exhibit very large errors and must be censored. The overlaid indicator arrays \((OV_V)\) and \((OV_W)\) are binary arrays where 0 indicates “not overlaid” and 1 indicates “overlaid”.

If \( n < N_2 - N_1 \)

(Segment I: Range gates that may or may not have overlaid echoes)

(Check power ratio using velocity threshold)
If $P_H(n) > P_H(n + N_1) \ 10^{0.1 T_{ow}}$
$OV_V(n) = 0$
Else
($Power \ ratio \ not \ met, \ but \ consider \ non-significant \ returns \ as \ non-existent$)
If $NS_V(n + N_1) = 1$
$OV_V(n) = 0$
Else
$OV_V(n) = 1$
End
End

(Check power ratio using width threshold)
If $P_H(n) > P_H(n + N_1) \ 10^{0.1 T_{ow}}$
$OV_W(n) = 0$
Else
($Power \ ratio \ not \ met, \ but \ consider \ non-significant \ returns \ as \ non-existent$)
If $NS_W(n + N_1) = 1$
$OV_W(n) = 0$
Else
$OV_W(n) = 1$
End
End
ElseIf $n < N_1$
($Segment \ II: \ Range \ gates \ that, \ based \ on \ the \ assumptions, \ never \ have \ overlaid \ echoes$)
$OV_V(n) = 0$
$OV_W(n) = 0$
Else
($Segment \ III: \ Range \ gates \ that \ may \ or \ may \ not \ have \ overlaid \ echoes$)
(Check power ratio using velocity threshold)
If $P_H(n) > P_H(n - N_1) \ 10^{0.1 T_{ow}}$
$OV_V(n) = 0$
Else
($Power \ ratio \ not \ met, \ but \ consider \ non-significant \ returns \ as \ non-existent$)
If $NS_V(n - N_1) = 1$
$OV_V(n) = 0$
Else
$OV_V(n) = 1$
End
End

(Check power ratio using width threshold)
If $P_H(n) > P_H(n - N_1) \ 10^{0.1 T_{ow}}$
$OV_W(n) = 0$
Else
($Power \ ratio \ not \ met, \ but \ consider \ non-significant \ returns \ as \ non-existent$)
If $NS_W(n - N_1) = 1$
$OV_W(n) = 0$
Else
$OV_W(n) = 1$
End
End
End

(Note that when processing the overlaid and significant return flags, the overlaid flags take a lower priority. That is, if a range bin is tagged as non-significant and also as overlaid, the overlaid indication is ignored and the gate is treated as a non-significant return only; e.g., painted black as opposed to purple)